Hinge-joined octahedra–rings, tubes, and hubs

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http://homepage.mac.com/whitby/

http://web.me.com/whitby/Octahedron/Welcome.html

References

Octahedron1stEd.pdf

Stabilization of rings of hinge-joined octahedra, bookmarks "QUASICRYSTAL, Twelvefold forms", page 138.
"NANOTUBE, Ring of sixteen edge joined octahedra", page 414
"VIRUS, page 393.

Introduction

Any number of identical regular octahedra can join edge to edge to form a ring. Each of the joins is a hinge and each hinge is parallel to each of the other hinges in the ring. This arrangement allows movement between the pair of planes defined by those edges of the octahedra which are perpendicular to the joined edges. This document looks at how the rings can be made structurally stable by preventing the octahedra from moving about their hinged joins [Fig. 1].

The stabilization results in rings of different symmetries within the same structure [Fig. 7]. Each octahedron within each ring has the potential to be part of an extended octahedrally crystalline formation. Identical rings can be joined edge to edge parallel to the axis of symmetry to produce tubes [Figs. 8 & 9]. An octahedron of a ring of one structure can be joined in an octahedrally crystalline assembly with an octahedron of a ring of another structure [Figs. 16, 17, 18]. In this manner, a complex polysymmetric structure consisting of linked hubs can be assembled.

Edgially joined octahedra can assemble as a sheet [Fig. 10] The sheet can roll up about one of two perpendicular axes so that the octahedra along opposite edges join to form a tube. Alternatively, the sheet could roll up in a spiral without joins.

The viral capsids show that a simple octahedron can represent a protein molecule of many residues. The simple octahedra of the structures shown herein may represent a similar complexity.

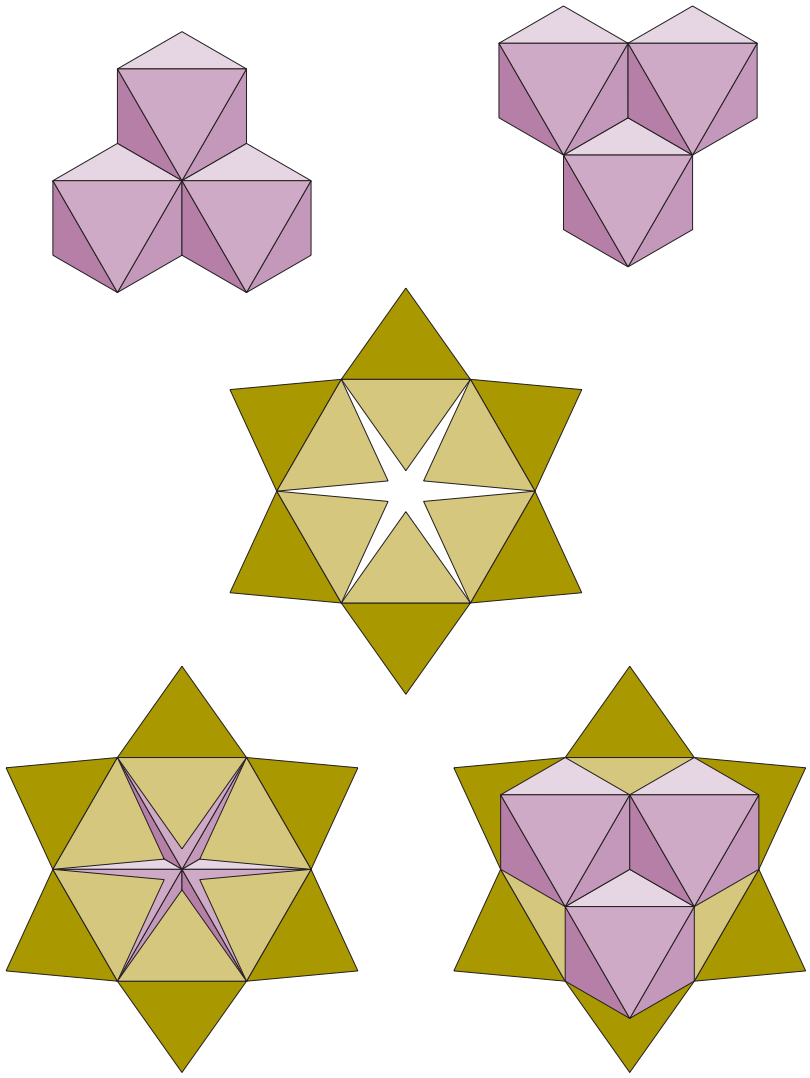

Fig 1–Stabilizing a ring of six edge joined octahedra

The figure shows how an octahedral triplet can stabilize a ring of six hinge joined octahedra. The unstabilized ring is shown in the center. Two triplets are shown in the top row. The ring at the bottom left is stabilized with the triplet shown on the top left which is visible through the gaps between the octahedra of the ring. The ring at the right is stabilized by the triplet from the top right.

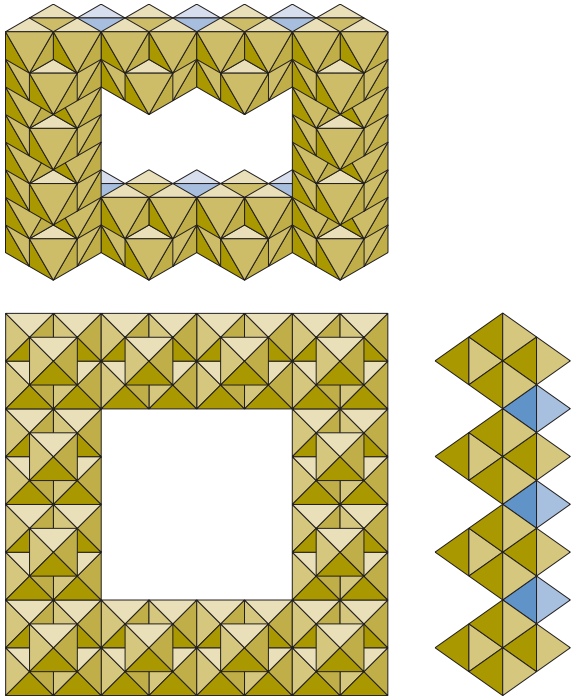

Fig 2–Square ring of compound octahedra stabilized by simple octahedra

The square ring of octahedra shown in the figure is composed of twelve compound octahedra. Each compound octahedron is composed of six identical simple octahedra. The ring is viewed from three directions. At lower left, the ring is viewed along a direction which is parallel to the axis of fourfold symmetry of the ring. On the lower right, the ring is parallel to the plane of the ring. The yellow octahedra belong to the compound octahedra; the blue belong to the simple octahedra which stabilize the ring. At the top, the ring is viewed so that the octahedra are facially projected.

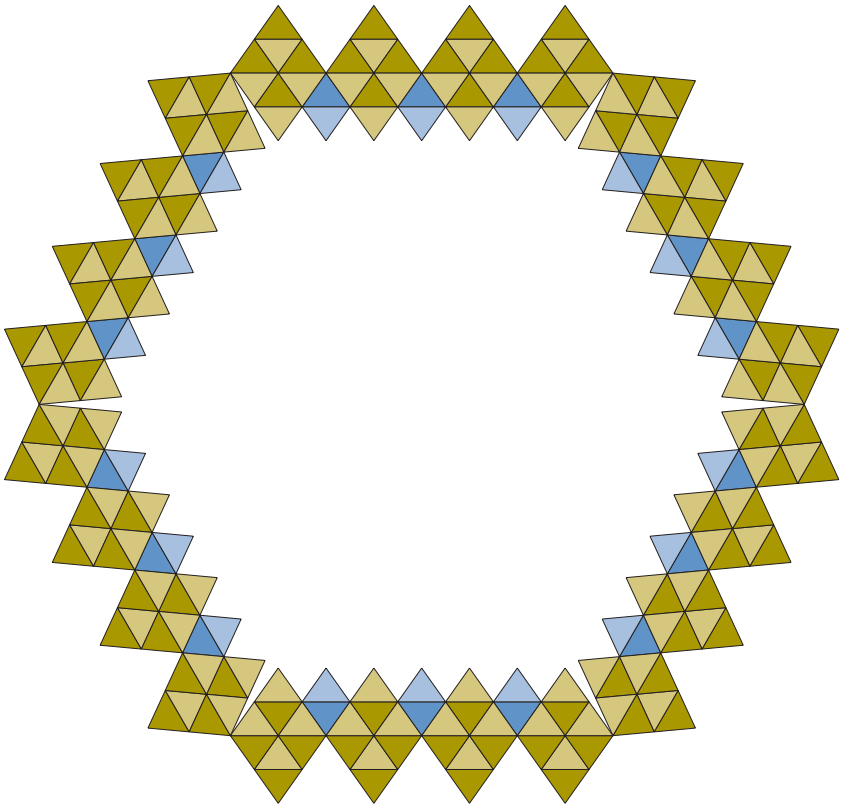

Fig 3–Ring of six edgially joined square octahedral rings

Six square rings identical to the one shown in the previous figure are joined edge to edge to form the unstabilized ring shown above.

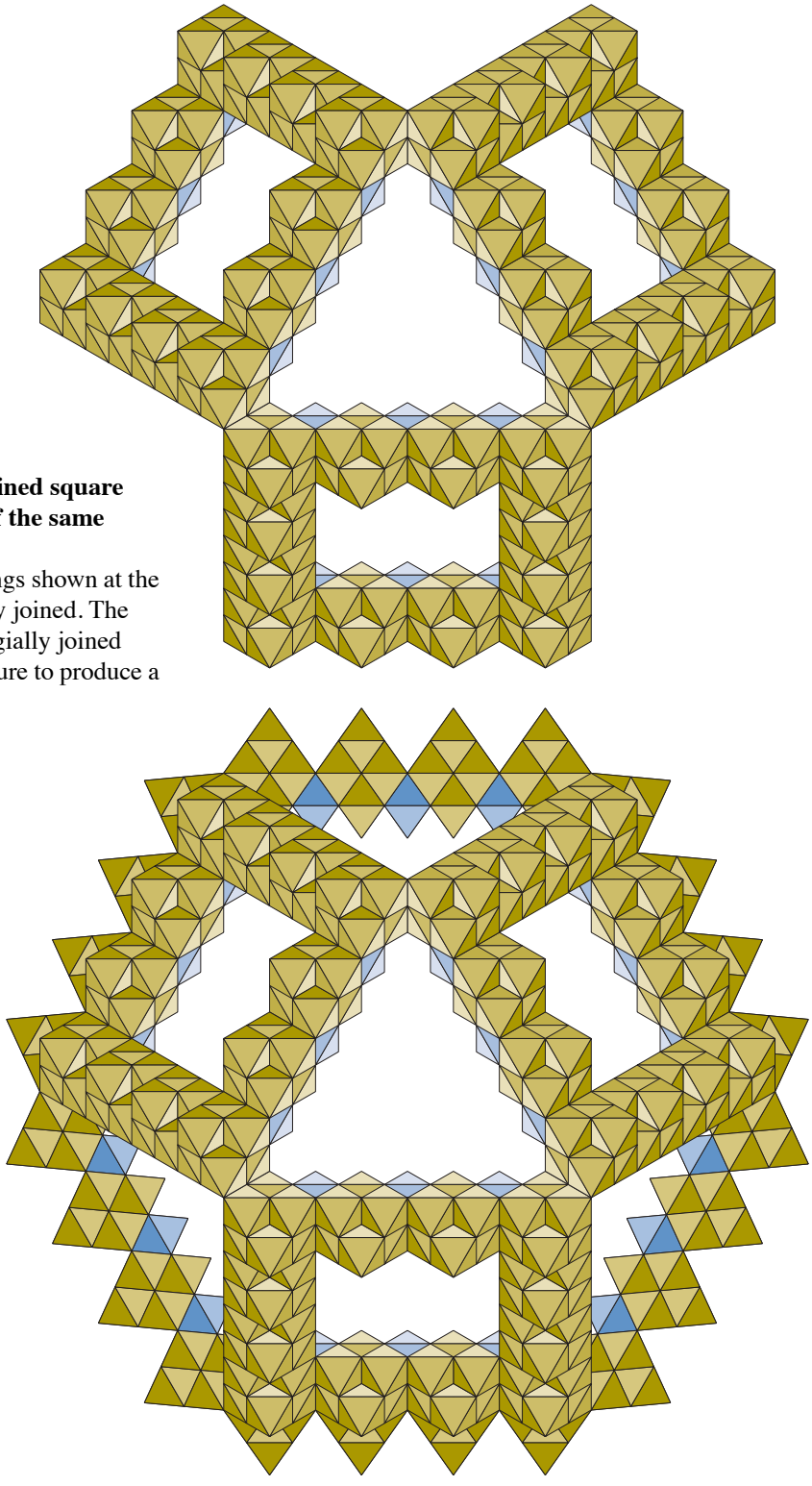

Fig 4–Ring of six hinged joined square rings stabilized by a triplet of the same square rings

The triplet of three square rings shown at the top of the figure are structurally joined. The triplet is joined with the six edgially joined square rings of the previous figure to produce a structurally stable assembly.

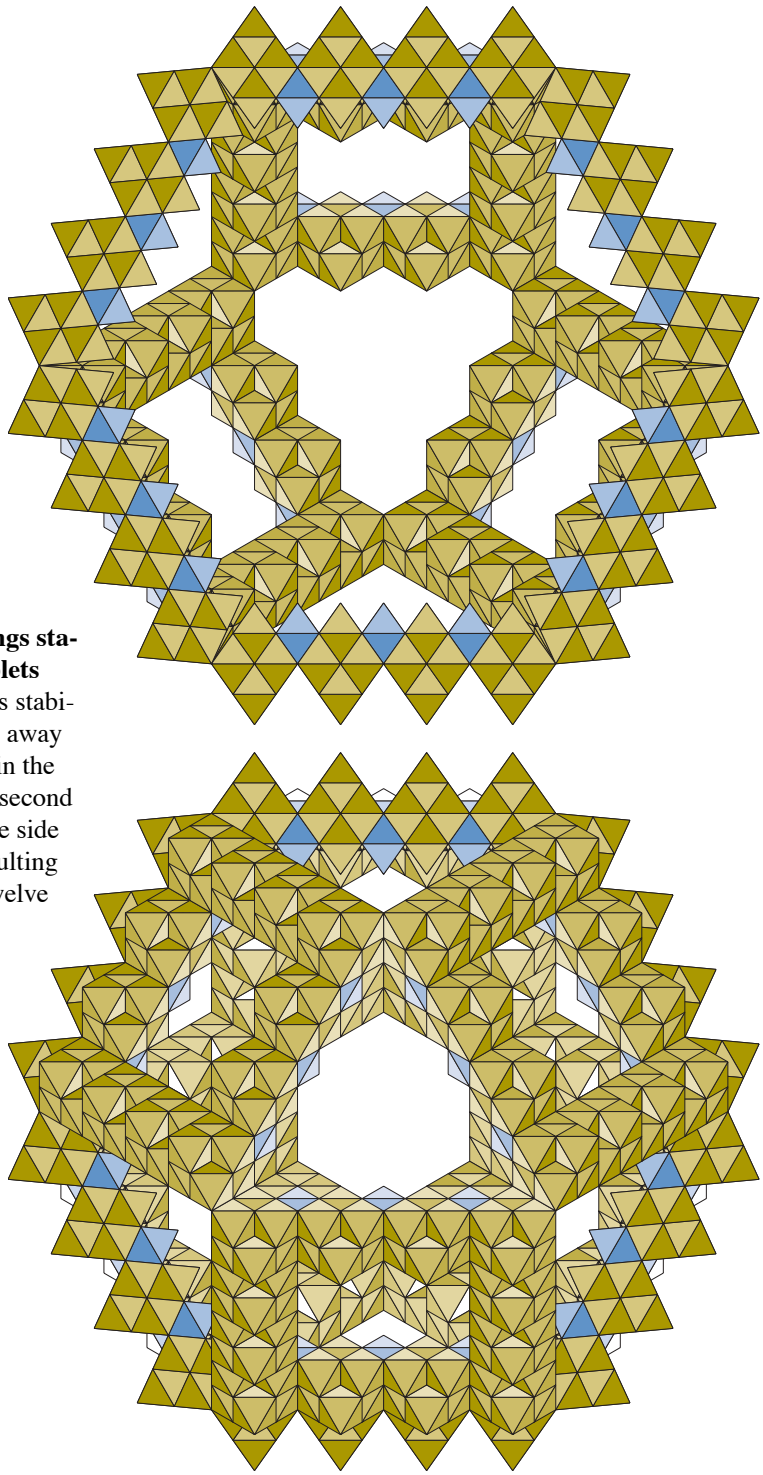

Fig 5–Ring of square rings stabilized by pair of ring triplets

The ring of square rings is stabilized by a triplet on the side away from the viewing direction in the top figure. At the bottom, a second triplet has been added on the side towards the viewer. The resulting assembly is composed of twelve identical square rings.

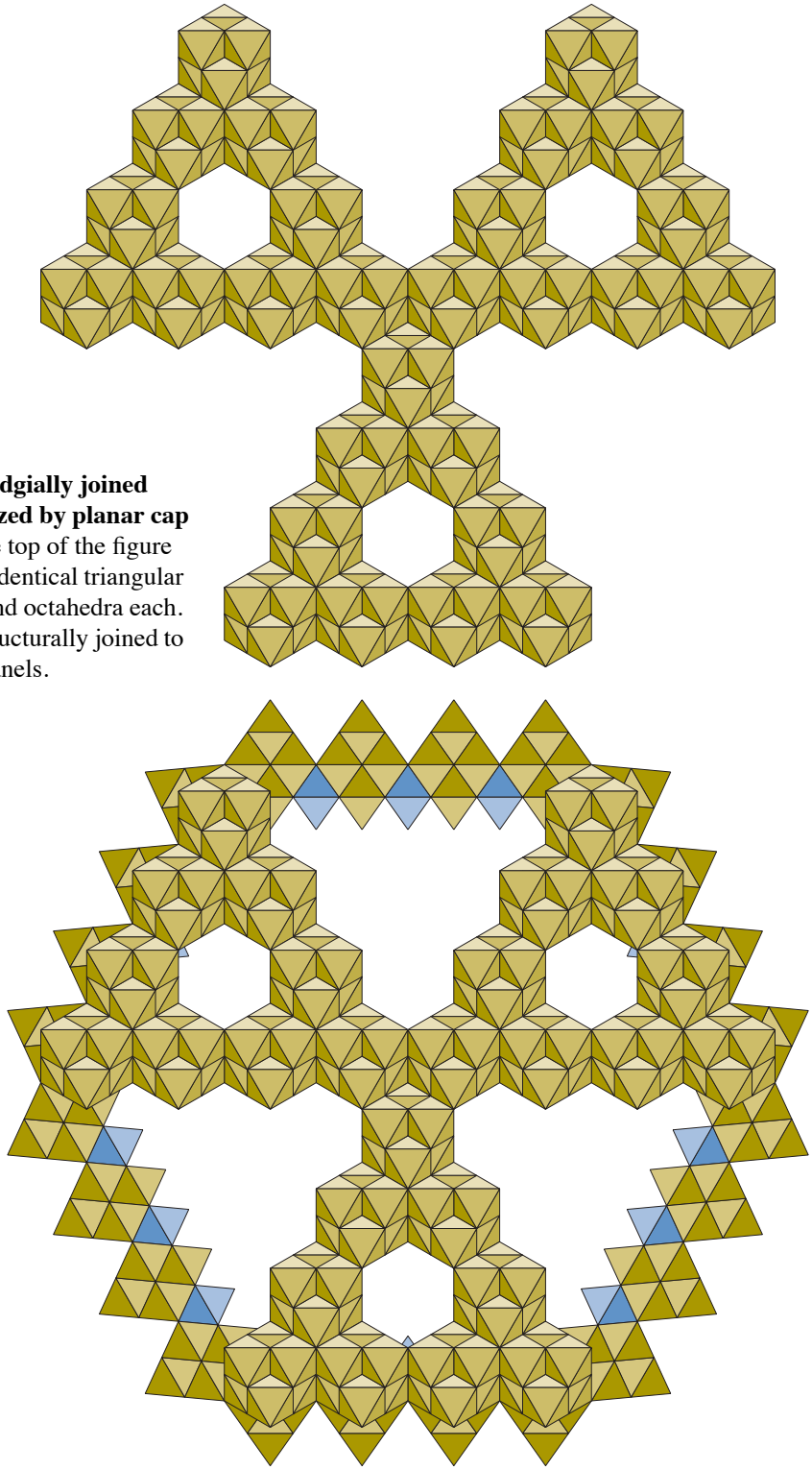

Fig 6–Ring of six edgially joined square panels stabilized by planar cap

The assembly at the top of the figure is composed of three identical triangular rings of nine compound octahedra each. Below it, the cap is structurally joined to a ring of six square panels.

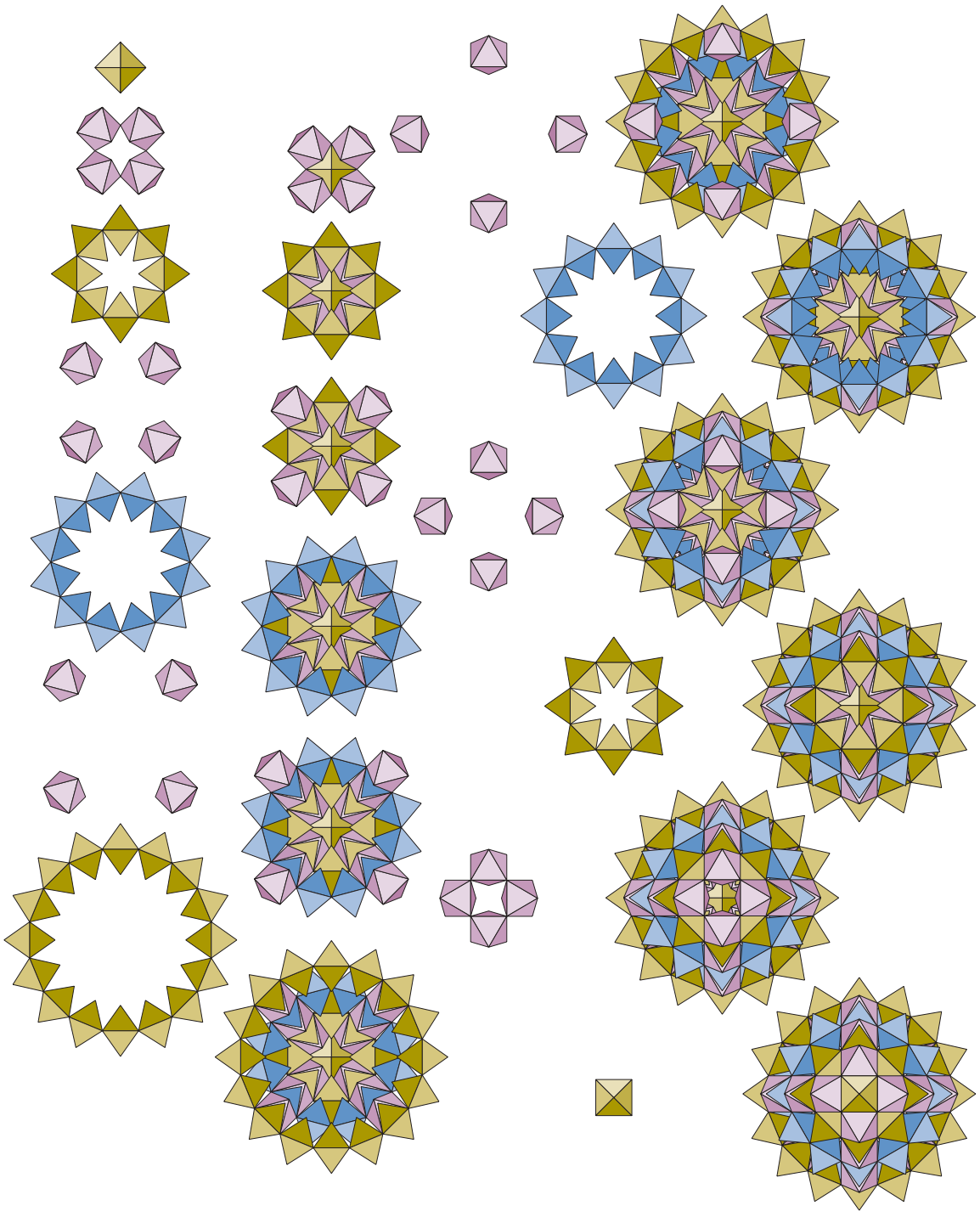

Fig 7–Assembly of five rings of edgially joined octahedra

The assembly begins at the top left and proceeds to the bottom, then returns to the top right and continues to the bottom. The growth direction is towards the viewer. The components are shown to the left of the growing assembly. The violet octahedra link the rings which are composed of yellow or blue octahedra. To stabilize the middle ring, the assemblies above the middle are rotated 45 degrees to those below the middle.

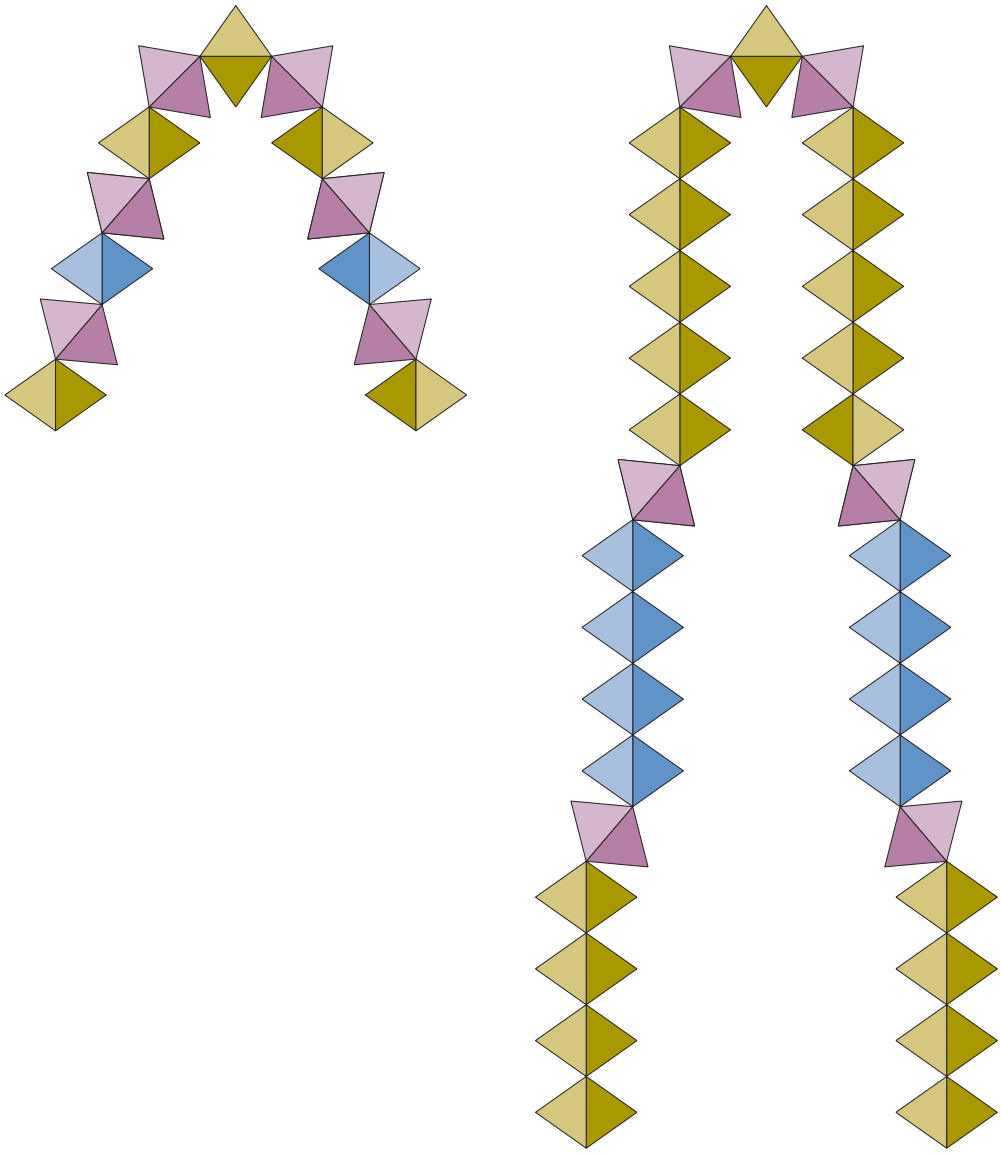

Fig 8–Assembly of five rings of hinge joined octahedra–stabilizing inter-ring linkages

The figure on the left shows the relationships between the octahedra of three of the rings and the octahedral links which join ring to ring. The view is normal to the axis of the assembly. Each octahedron is bisected by a plane which includes the vertexial axis parallel to the viewing plane. The plane also includes the axis of the assembly. The two yellow octas at the bottom are diametrically opposite components of the 16-ring. The two blue octas are diametrically opposite components of the 12-ring. The two yellow octas above are diametrically opposite components of the 8-ring. The violet octas link the rings to one another and to the stabilizing octahedron at the top.

The figure on the right shows how identical rings can be added to form concentric regular polygonal cylinders within the same structure.

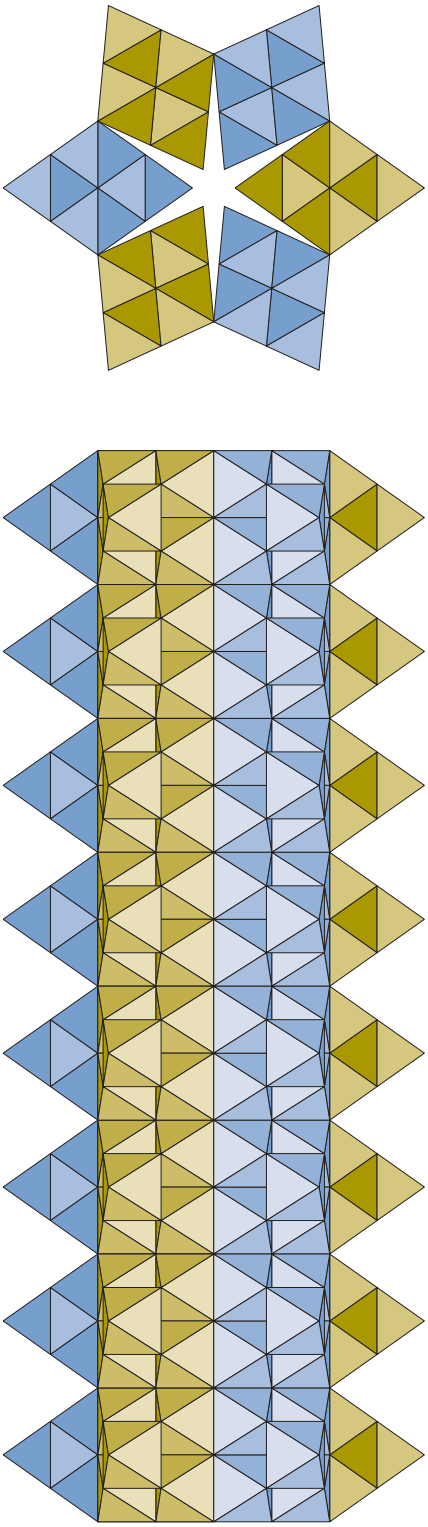

Fig 9–Cylinder composed of rings of six hinge joined compound octahedra

The figure shows a cylinder composed of eight identical hinge joined octahedral rings. Each compound octahedron is composed of six simple octahedra. The view at the top of the figure is parallel to the axis of the cylinder. The view below it is perpendicular to the cylindrical axis.

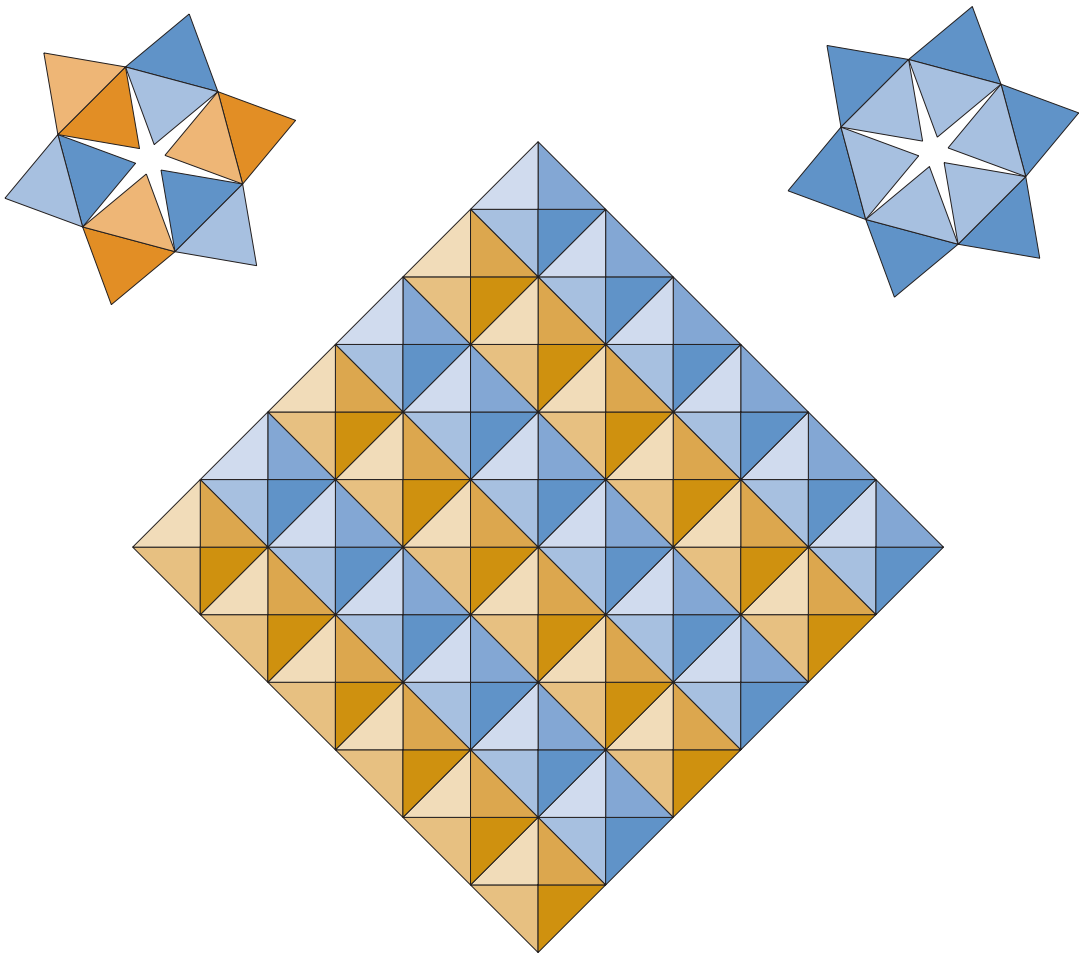

Fig 10–Square 6x6-layer of edgially joined regular octahedra

The 6x6-layer of regular octahedra can form either of the two cylinders shown at the top. The alternating orange and blue ring is the end view of the cylinder which results when the six blue octas along the upper right of the layer join edge-to-edge with the six orange octas along the lower left. The blue ring at the top right is the axial view of the cylinder which results when the blue and orange octas of the upper left join edge-to-edge with their like colored counterparts of the lower right.

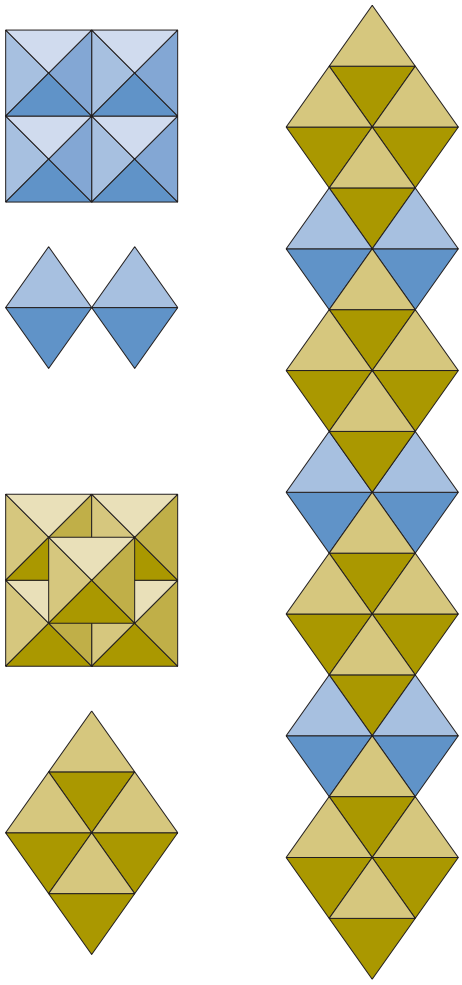

Fig 11–Assembly of a square octahedral prism

The figure shows the assembly of a square octahedral prism. The prism consists of five compound octahedra composed of six yellow octahedra each which are joined together by a square array of four octahedra colored blue. The prism is shown on the right; the components are shown in both axial and radial views on the left.

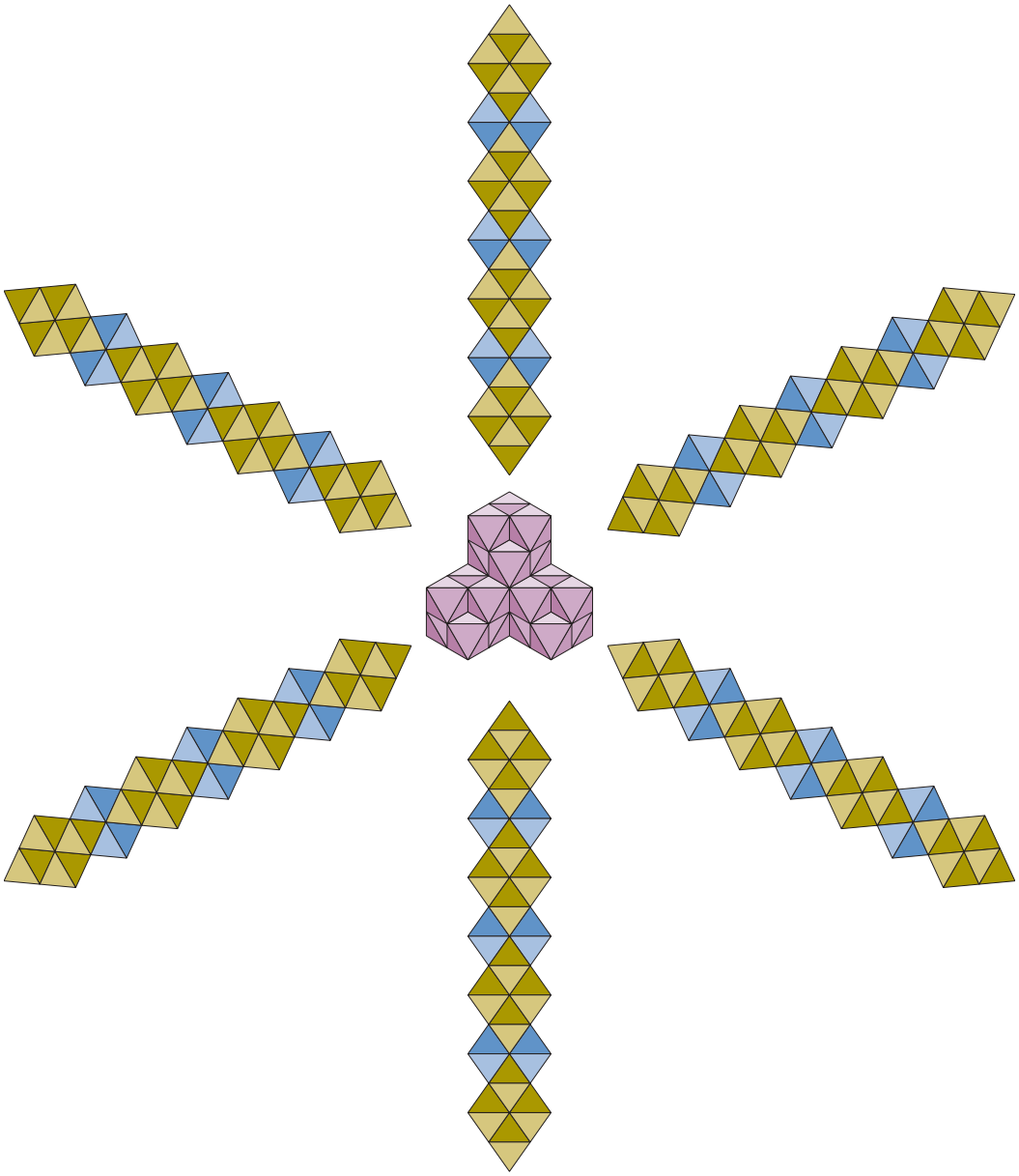

Fig 12–Six square octahedral prisms and an octahedral triplet

Six prisms identical to the one shown in the previous figure are arrayed here with an octahedral triplet. The angle between each pair of adjacent prisms is 60-degrees.

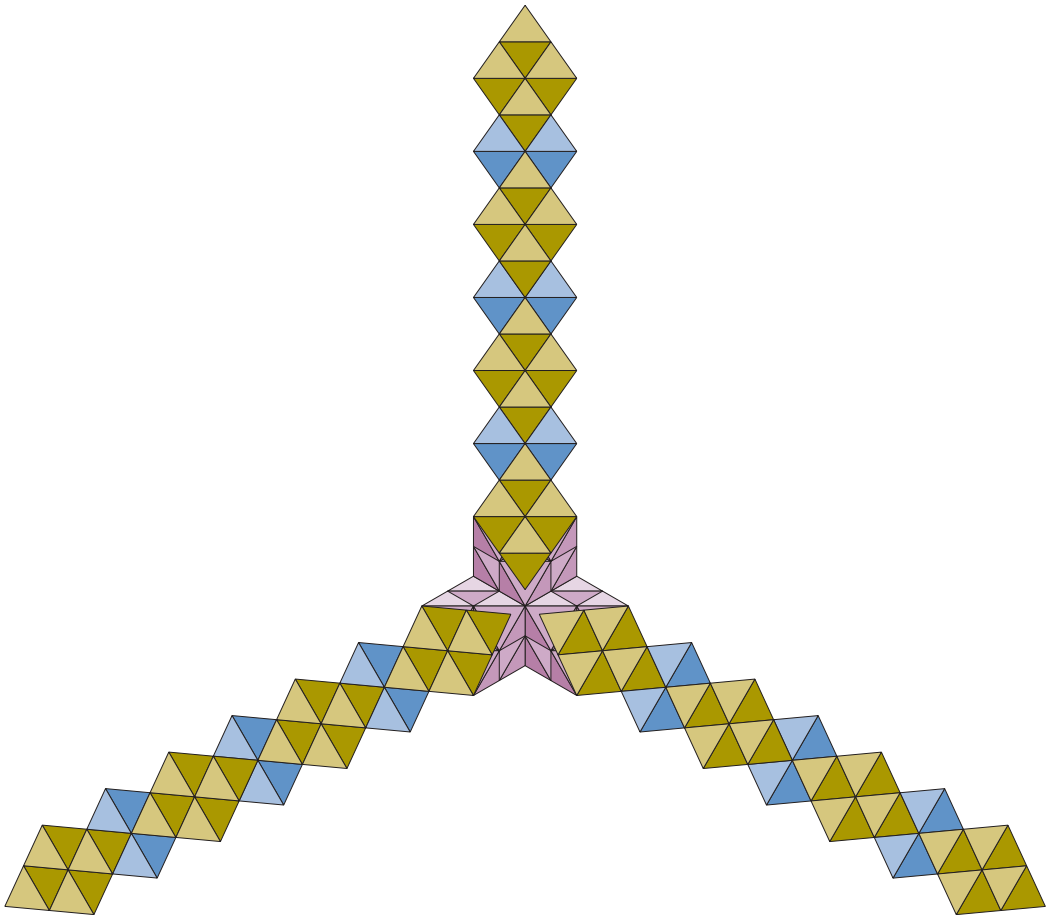

Fig 13–Three square octahedral prisms hinge joined to an octahedral triplet
Three identical prisms which differ by an angle of 120-degrees from each of their neighbors are hinge joined to a triplet of three compound octahedra. Each prism can rotate about its hinge join with the triplet independently of the other prisms.

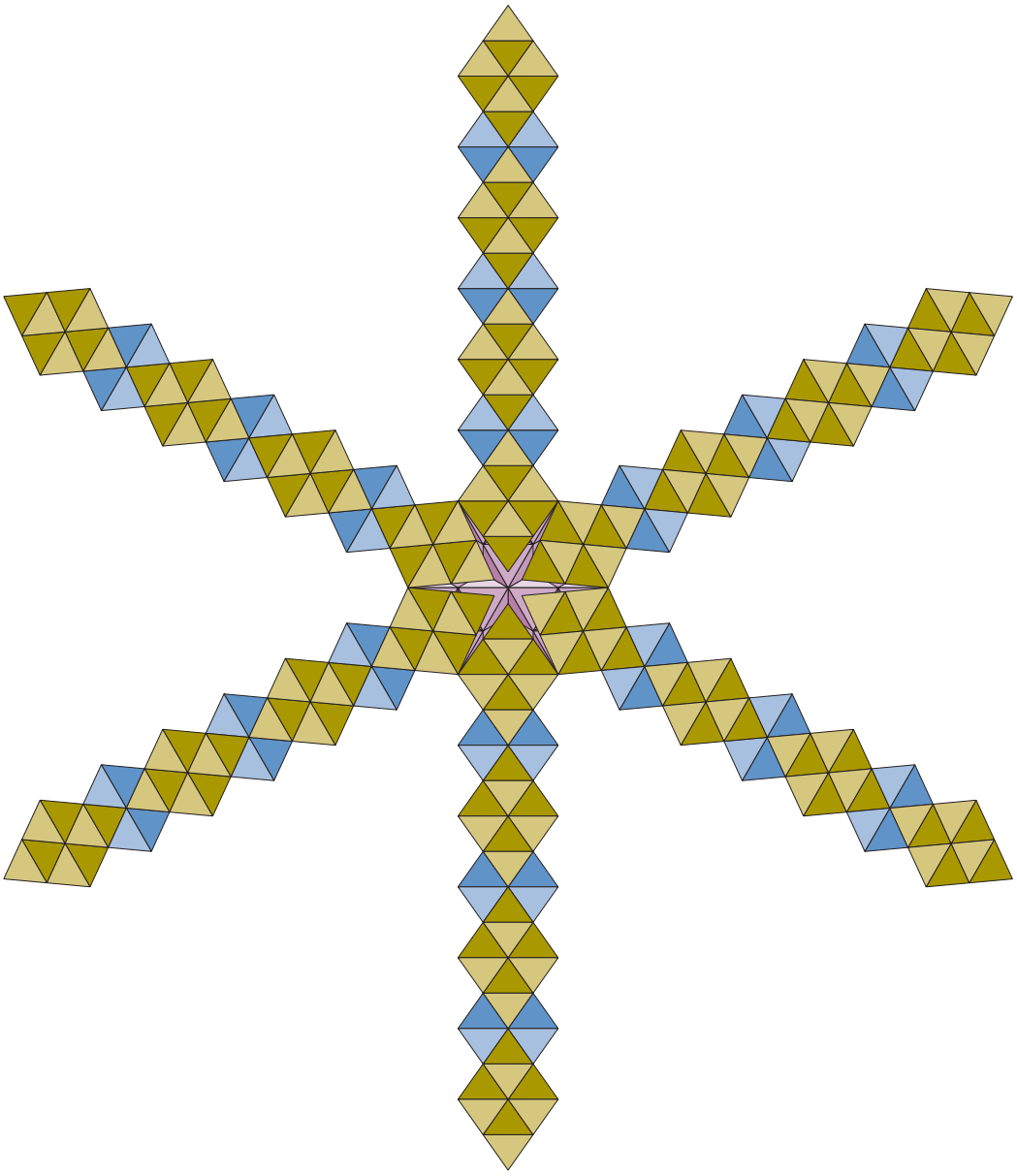

Fig 14–Ring of six hinge joined prisms stabilized by an octahedral triplet

The addition of three more prisms to the previous figure stabilizes the assembly. The prisms form a hinged ring which permits the prisms to move within the plane of the ring only. The triplet prevents this motion by fixing every other prism of the ring. These, in turn, fix the other three prisms.

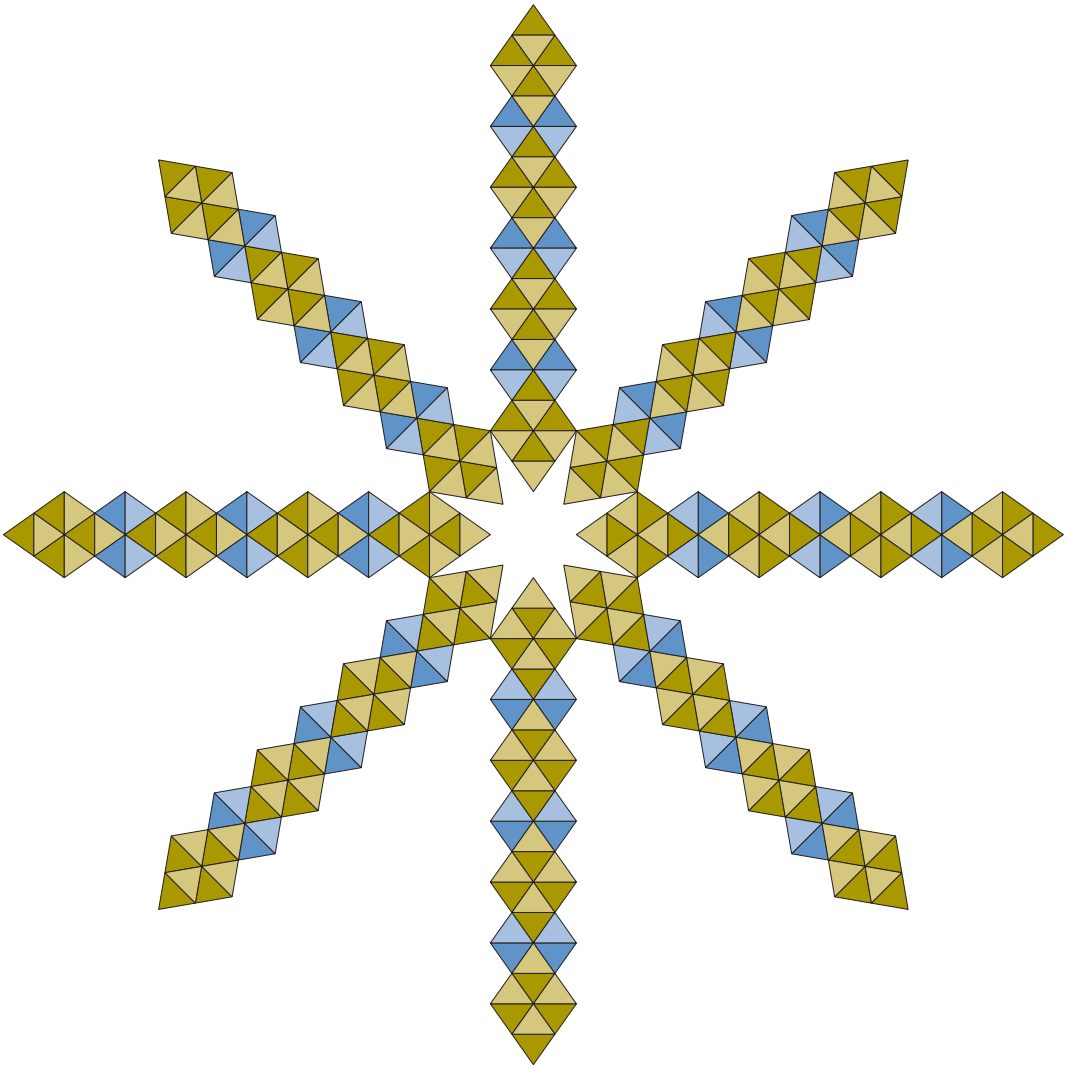

Fig 15–Ring of eight hinge joined prisms

The eight identical prisms are hinge joined to form a ring. The axis of each prism is 45-degrees from the axis of each of its neighbors. This ring can be stabilized in the manner shown in the axially symmetric assembly.

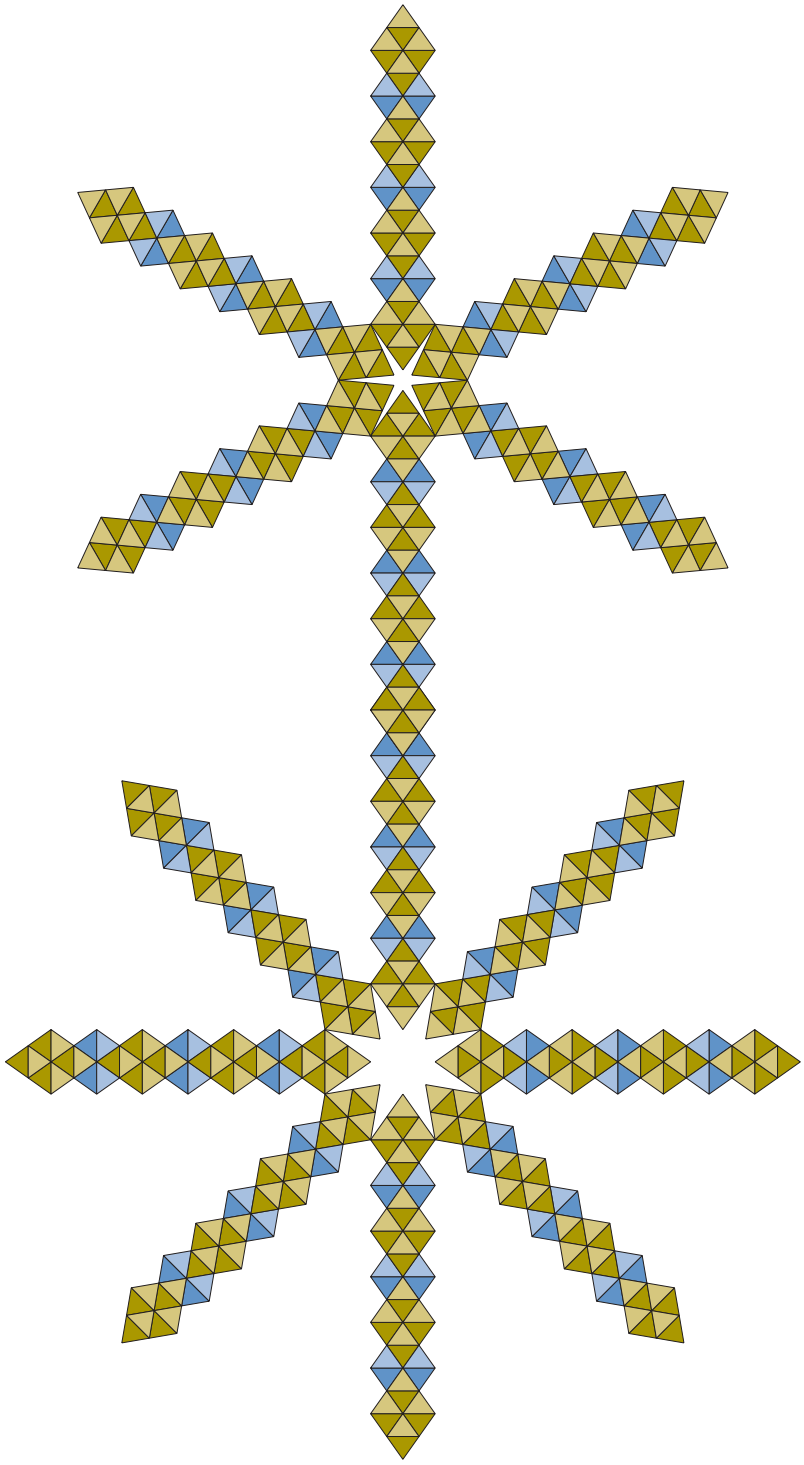

Fig 16–Prism to prism connection of six ring with eight ring

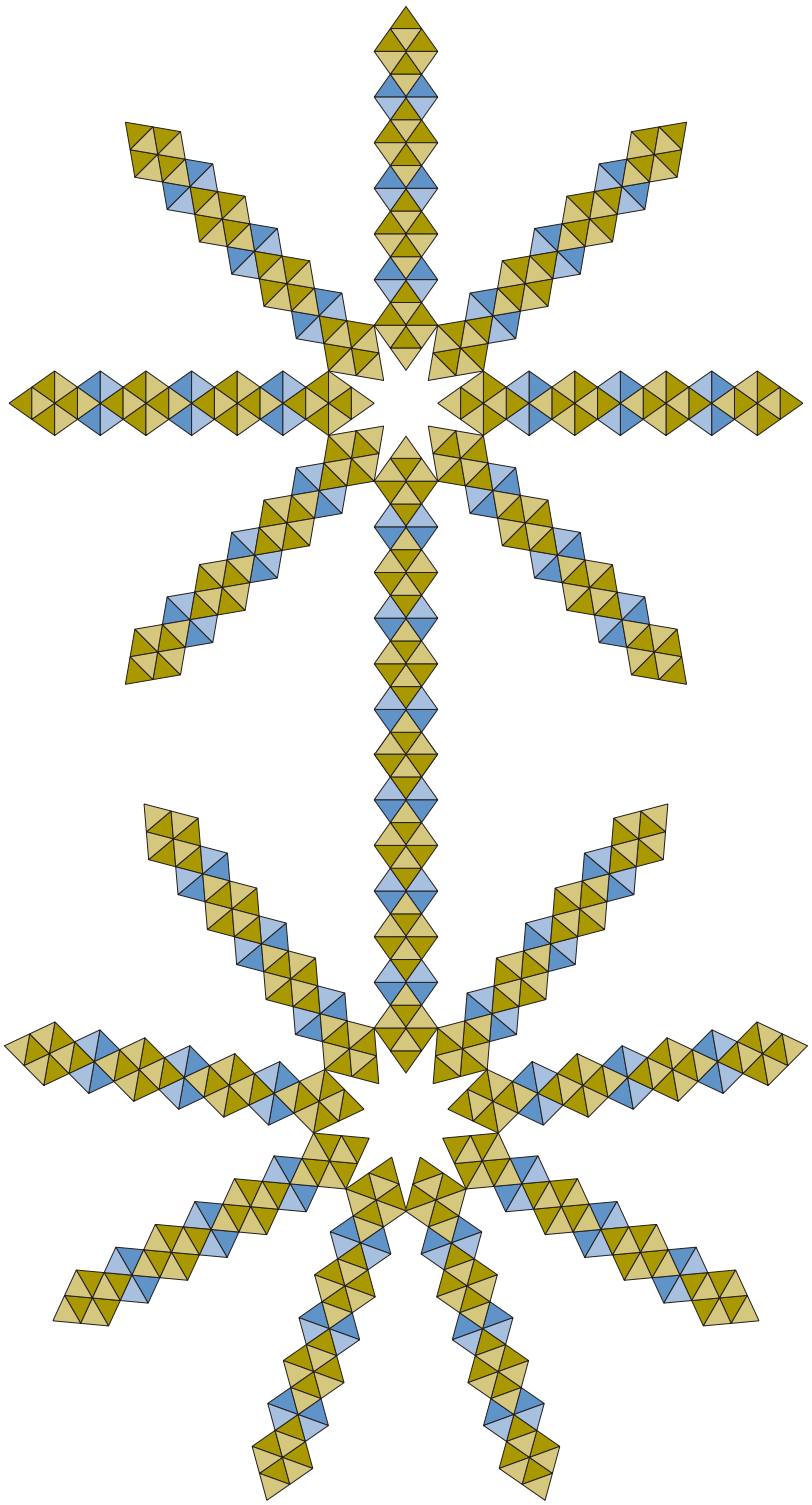

Fig 17–Prism to prism connection of eight ring with nine ring

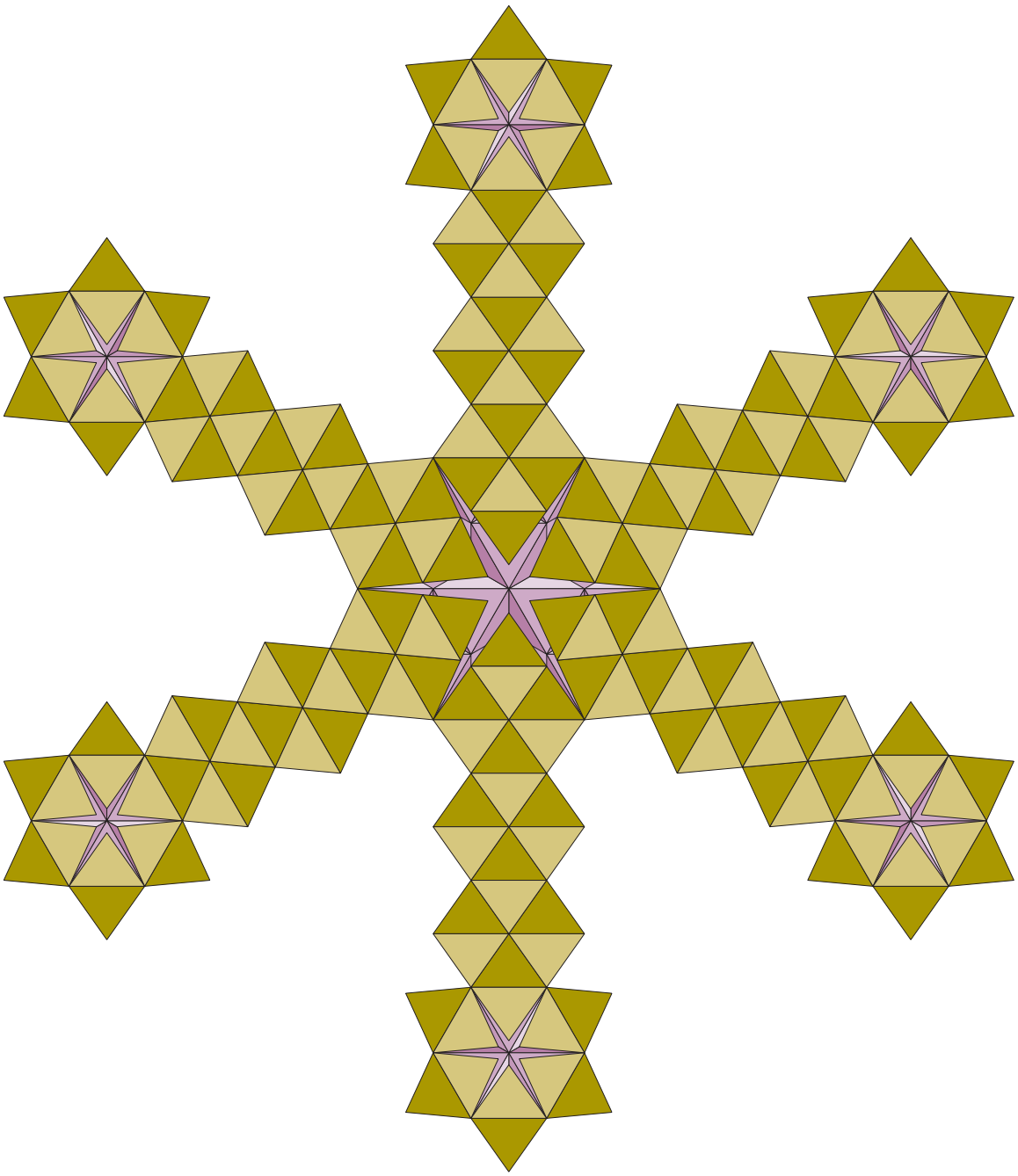

Fig 18–Ring of six prisms stabilized by a triplet of compound octahedra and terminated by a ring of six simple octahedra stabilized by a triplet of simple octahedra.

The figure shows how rings of hinge joined regular octahedra can form networks with other rings to produce complex polycrystalline assemblies.