Structural assemblies of hinge-joined regular hexagonal panels

Robert William Whitby
27 April 2007
http://homepage.mac.com/whitby/

http://web.me.com/whitby/Octahedron/Welcome.html

Reference

1. Expo 67 in Montreal–A photo collection about Canada's Centennial Celebration!

This URL accesses the index of the photo collection with links to each of the pavilions.
http://expo67.ncf.ca/

2. Man the Producer pavilion

http://expo67.ncf.ca/man_the_producer_p1.html

3. Man the Explorer pavilion

http://expo67.ncf.ca/expo_thematic_p1.html

4. Expo Architecture: Theme Pavilions

Relates the architectural intent to build truncated tetrahedral buildings using the form of the truncated tetrahedron as a design unit.
http://expolounge.blogspot.com/2006/07/expo-architecture-theme-pavilions.html

5. Octahedron, the Universe defined by Robert William Whitby

A description of the atomic shapes and how they join which follows from the discovery that the periodicity of the atomic elements matches the periodicity of recurring form in which identical regular octahedra combine to form ever larger regular octahedra. Octahedron1stEd.pdf shows that the atomic elements are crystalline assemblies of identical regular octahedra and explores the implications of this discovery. 500 pages. See Octahedron1stEd.pdf at the following URL.
http://homepage.mac.com/whitby/FileSharing103.html

6. Icosahedral assemblies of triangular panels of diamond CFUs by Robert William Whitby

Download the file DiamondPanels.pdf at the following URL.
http://homepage.mac.com/whitby/Quasicrystals/FileSharing176.html

7. Polyhedral assemblies of truncated triangular panels of graphite & diamond by Robert William Whitby

Download the file TruncPanels.pdf at the following URL.
http://homepage.mac.com/whitby/Quasicrystals/FileSharing180.html

8. Diamondoids by Robert William Whitby

Download the file Diamondoids.pdf at the following URL.
http://homepage.mac.com/whitby/Quasicrystals/FileSharing186.html

9. Diamondoid-cyclohexamantane by Robert William Whitby

Download the file Cyclohexamantane.pdf at the following URL–
http://homepage.mac.com/whitby/Quasicrystals/FileSharing187.html

Figures

Introduction

Expo67

This paper examines structures of hinge-joined regular hexagonal panels. It is inspired by the designer's intent to use truncated tetrahedral building blocks to produce truncated tetrahedral buildings to house the Theme Pavilions of Expo67 in Montreal, Quebec, Canada. [See Reference 4.] Figure 1 shows the relationship of the truncated tetrahedron to the regular tetrahedron. Figures 2, 3, and 4 are photographs showing exterior views of truncated tetrahedral buildings used in the Man the Producer pavilion at Expo67 in Montreal. [See References 1, 2, and 3.]

The diamond CFU

The crystal forming unit of diamond consists of four C-atoms. Each atom acts as a triangular panel. The four panels enclose a regular tetrahedral volume. [See CARBON in Reference 5.] Figure 5 shows the CFU as part of a tetrahedral assembly of He-octas; Figure 6 shows the same tetrahedral assembly inverted. A CFU within a diamond crystal is joined to four identical CFUs. Figure 7 shows that for a diamond CFU to join with four identical CFUs that have been incorporated as part of identical tetrahedral assemblies of He-octas a vertexial He-octa must be removed. The space occupied by the vertexial He-octa is a regular tetrahedron. That space is shared equally by each of the four surrounding CFUs. The volume occupied by the CFU is a truncated tetrahedron with one-fourth of a tetrahedron mounted on each triangular face. [See Figure 8.]

Figure 9 shows how the contact between two truncated tetrahedral assemblies of He-octas defines a regular hexagon.

Figure 10 shows three truncated tetrahedral assemblies joined edge to edge in a triangular assebly. The same structure is shown using three identical truncated tetrahedral shapes.

Figure 11 shows how seven identical truncated tetrahedral shapes can form an hexagonal panel; and how four identical panels join together to enclose a truncated tetrahedral volume.

Figure 12 shows how four identical truncated tetrahedra can join edge to edge to form two different regular tetrahedral assemblies.

Structural assemblies of hexagonal panels

Figure 13 shows how three identical hexagonal panels can form a stable open faced truncated tetrahedral structure.

Figure 14 shows how identical hexagonal panels can make linear structures consisting of open faced truncated tetrahedral volumes.

Figure 15 is a photograph of two truncated tetrahedral assemblies made of identical hexagonal panels–a lone truncated tetrahedron consisting of four panels and three open faced truncated tetrahedral units joined edge to edge to form an inverted open faced truncated tetrahedral assembly.

Figures 16 and 17 show how identical hexagonal panels can join to form panels which mimic the structures that diamond CFUs are able to form that can be incorporated as the panels of fullerenes.

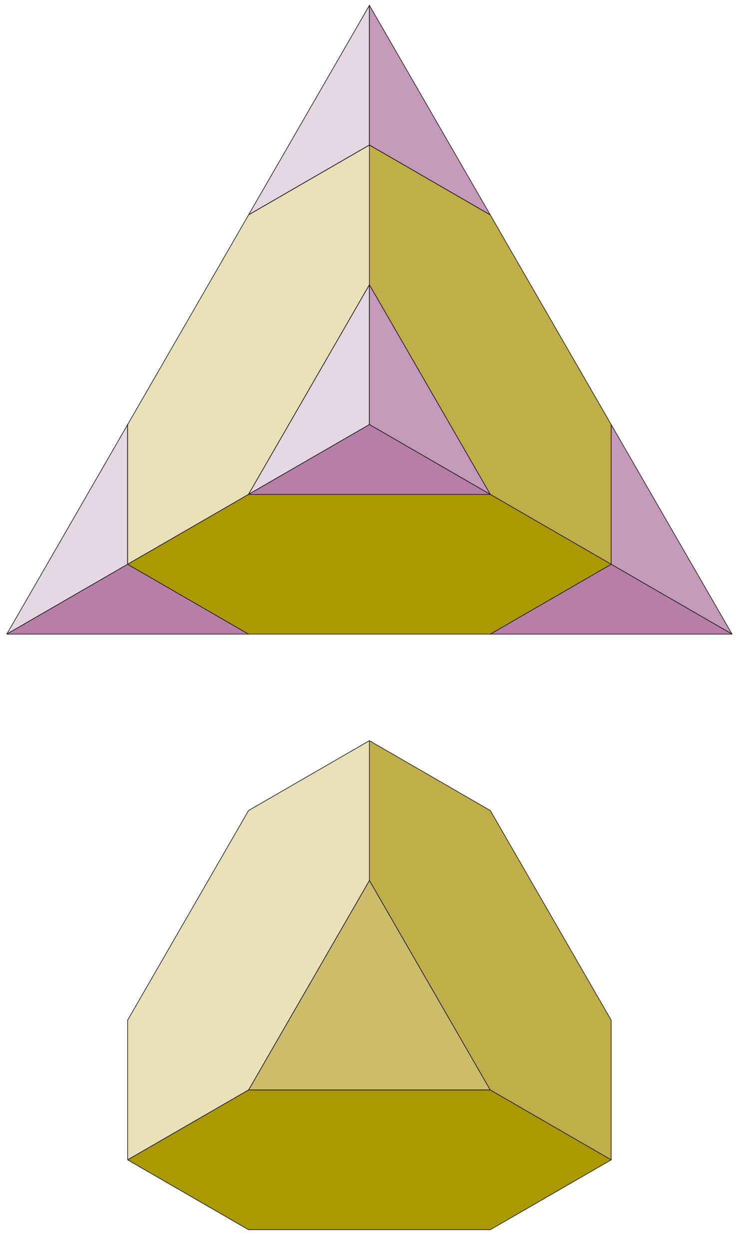

Fig. 1 Vertexially truncated regular tetrahedron

The figure shows how a regular tetrahedron can be vertexially truncated by removing a regular tetrahedron from each of its vertexes. The truncation chosen here leaves a polyhedron which has four regular hexagonal faces and four regular triangular faces. Each of the eight faces is parallel to one of the faces of z regular octahedron.

Fig. 2 Truncated tetrahedral buildings at Expo67
This photo shows two buildings each having the form of a truncated tetrahedron.

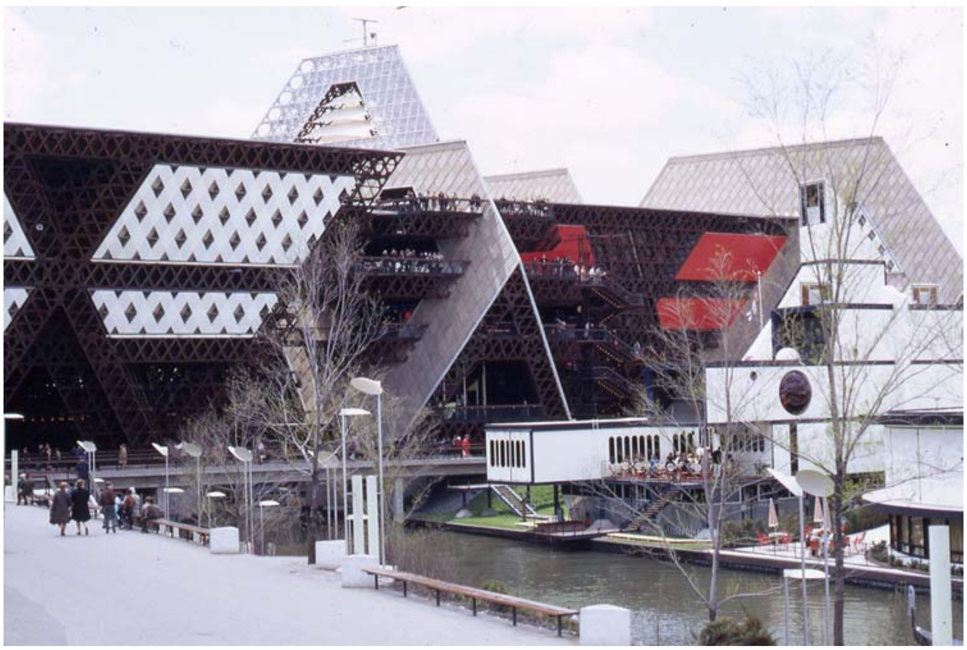

Fig. 3 Expo67 pavilion incorporating truncated tetrahedral structures

Fig. 4 Expo67 pavilion incorporating truncated tetrahedral structures–near view

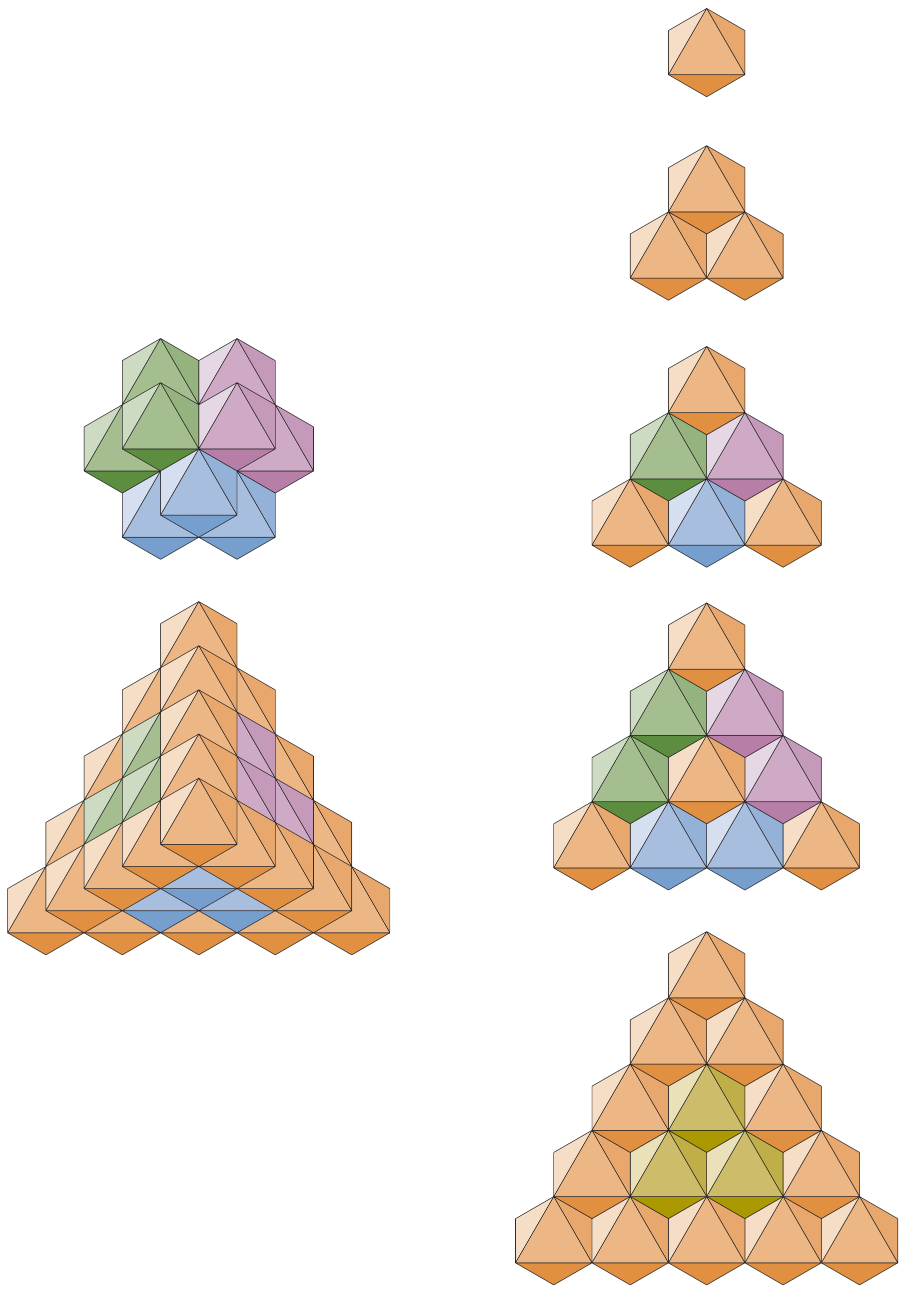

Fig. 5 The crystal forming unit of diamond

The figure shows how the crystal forming unit of diamond (CFU) fits within a regular tetrahedral assembly of He-octas. The CFU is shown at the top left. The tetrahedral assembly is shown on the bottom left. On the right, the locations of the He-octas of the four C-atoms of the CFU are shown in each of the three layers that include them.

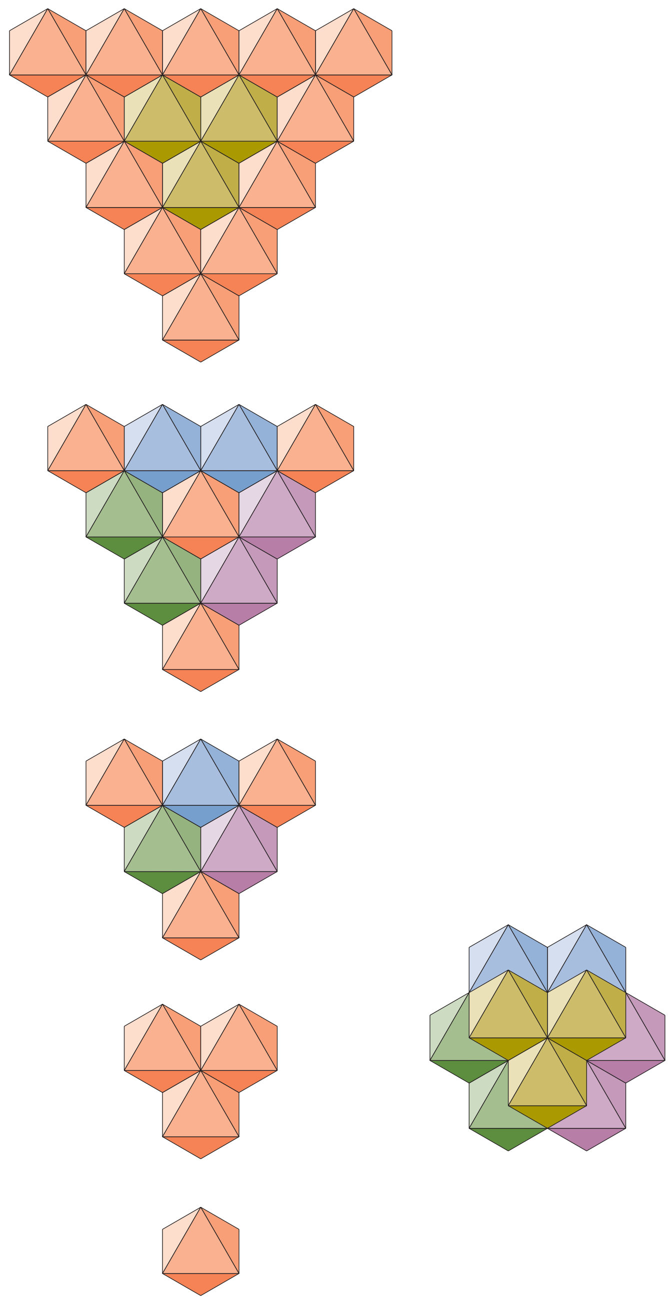

Fig. 6 The crystal forming unit of diamond–inverted
The figure shows each of the layers included in the tetrahedron which incorporates the CFU of diamond. The layer at top left is the near layer; the lone He-octa at bottom left is the far layer. The CFU is shown on the right.

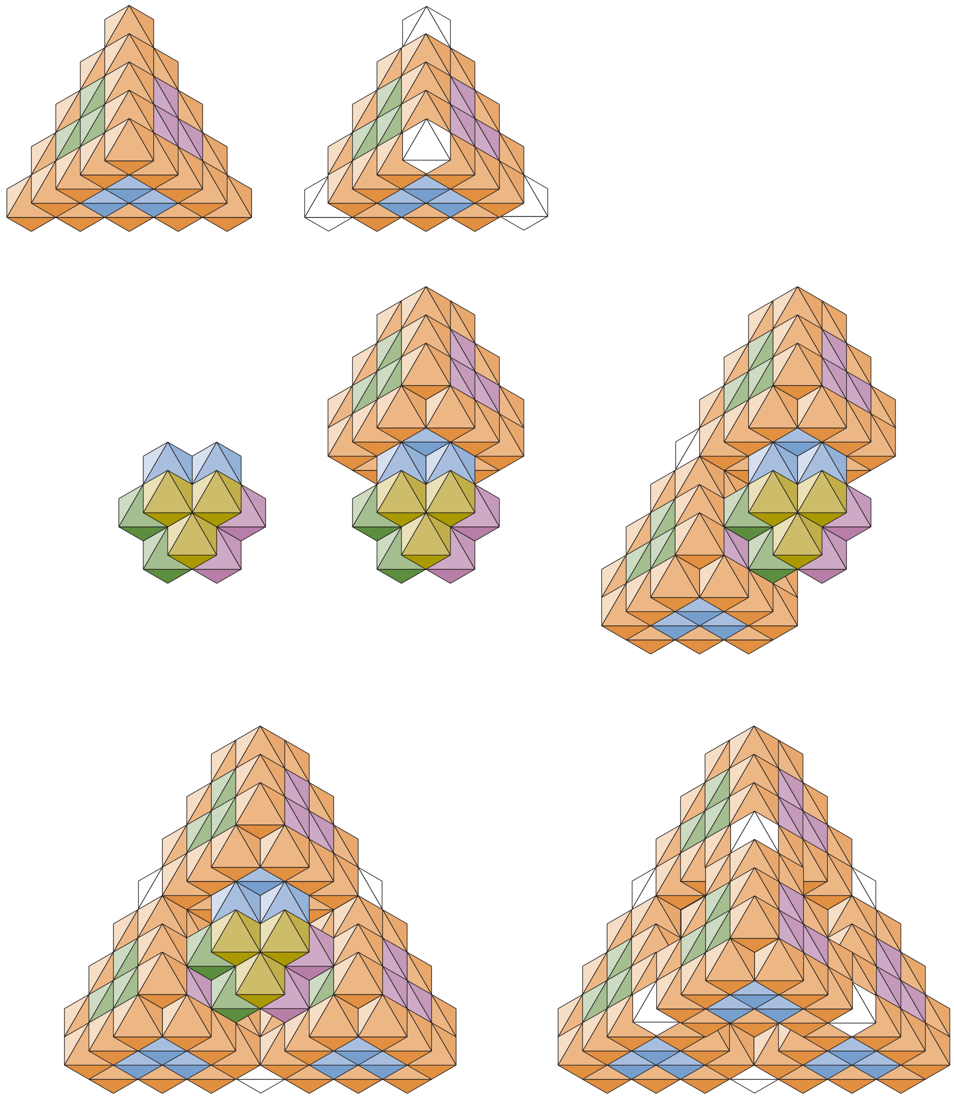

Fig. 7 Tetrahedral assembly of five diamond CFUs
The figure shows that the tetrahedral assembly of five diamond CFUs requires truncated tetrahedral volumes. *At top left*, the CFU is embedded in a tetrahedral assembly. *At top right*, the four vertexial He-octas have been removed leaving their locations marked with colorless octahedra. *At mid left*, the inverted CFU is shown bare. *To its right*, the inverted CFU is joined with a CFU embedded in a truncated tetrahedron. *At mid right*, a second embedded CFU is joined to the inverted CFU. The colorless octahedron at the junction of the truncated tetrahedral assemblies shows a vertexial location that is shared by each. *At bottom left*, a third embedded CFU is joined to the inverted CFU. A colorless octahedron occupies two vertexial locations shared by it and its two embedded neighbors. *At bottom right*, the fourth embedded CFU is joined to the inverted CFU to complete the tetrahedral assembly of five diamond CFUs. This embedded CFU shares a vertex with each of the other three embedded CFUs; the locations are marked with a colorless octahedron.

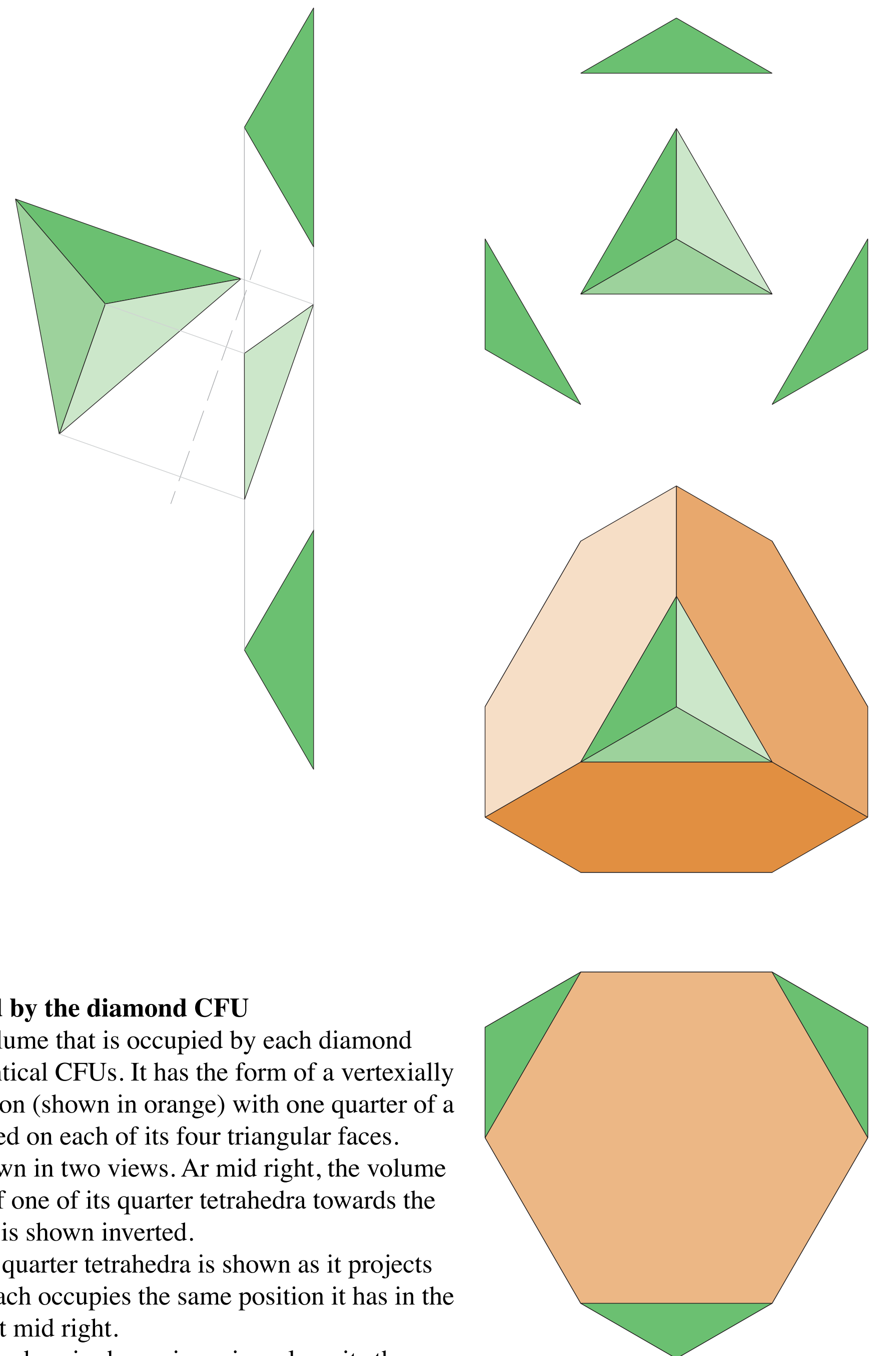

Fig. 8 Volume occupied by the diamond CFU

The figure shows the volume that is occupied by each diamond CFU that adjoins four identical CFUs. It has the form of a vertexially truncated regular tetrahedron (shown in orange) with one quarter of a regular tetrahedron mounted on each of its four triangular faces.

The CFU volume is shown in two views. Ar mid right, the volume is shown with the vertex of one of its quarter tetrahedra towards the viewer. At bottom right, It is shown inverted.

At top right, each of the quarter tetrahedra is shown as it projects upon the viewing plane. Each occupies the same position it has in the view of the CFU volume at mid right.

At left, the quarter tetrahedron is shown in a view along its three-fold axis. A second view that is perpendicular to the first is taken so that one edge of its regular triangular base is viewed end on. A third view is taken perpendicular to the second view so that the edge between two of its isosceles faces is seen end on and the third isosceles face projects as shown at the top. A fourth view taken in the diametrically opposite direction shows a projection of the base face. The third and fourth views result in identical projections.

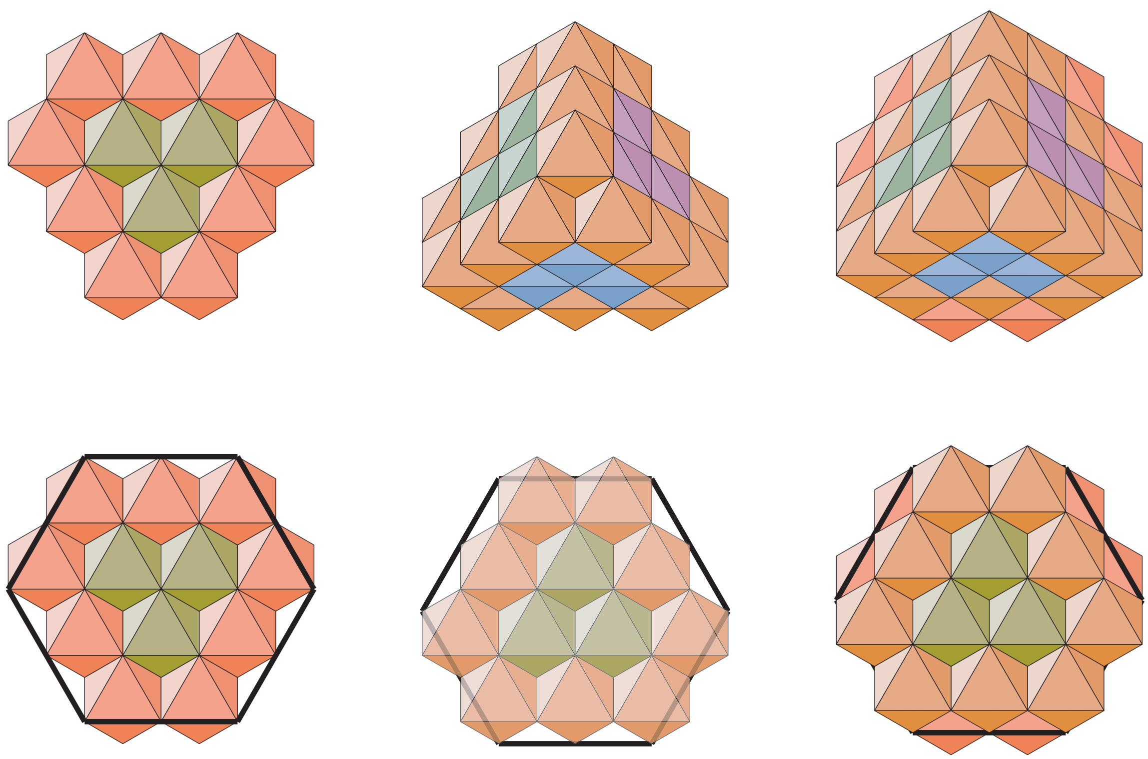

Fig. 9 The join between two embedded CFUs is regular hexagonal.
At top left, an inverted CFU embedded in a truncated tetrahedron is shown. At top mid, a second embedded CFU is shown. At top right, the two embedded CFUs are joined.
At bottom left, the joining layer of the inverted CFU is shown with the regular hexagon. At bottom mid, the joining layer of the other embedded CFU is shown with the regular hexagon. At bottom right, the two layers are joined and the hexagons are congruent.

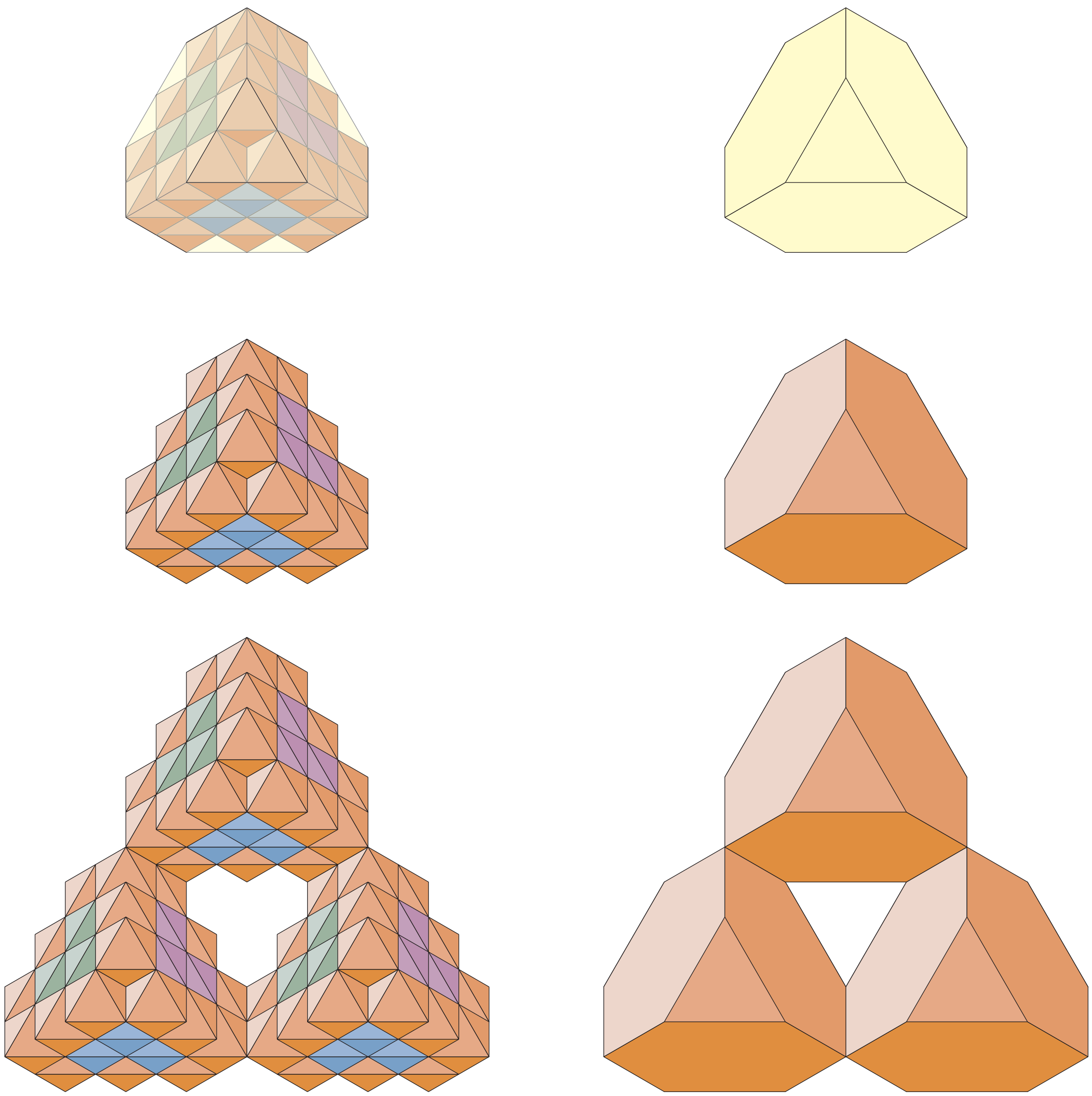

Fig. 10 Truncated tetrahedral assembly mimics diamond CFU assembly
The figure shows that the diamond structure can be mimiced using truncated tetrahedra.

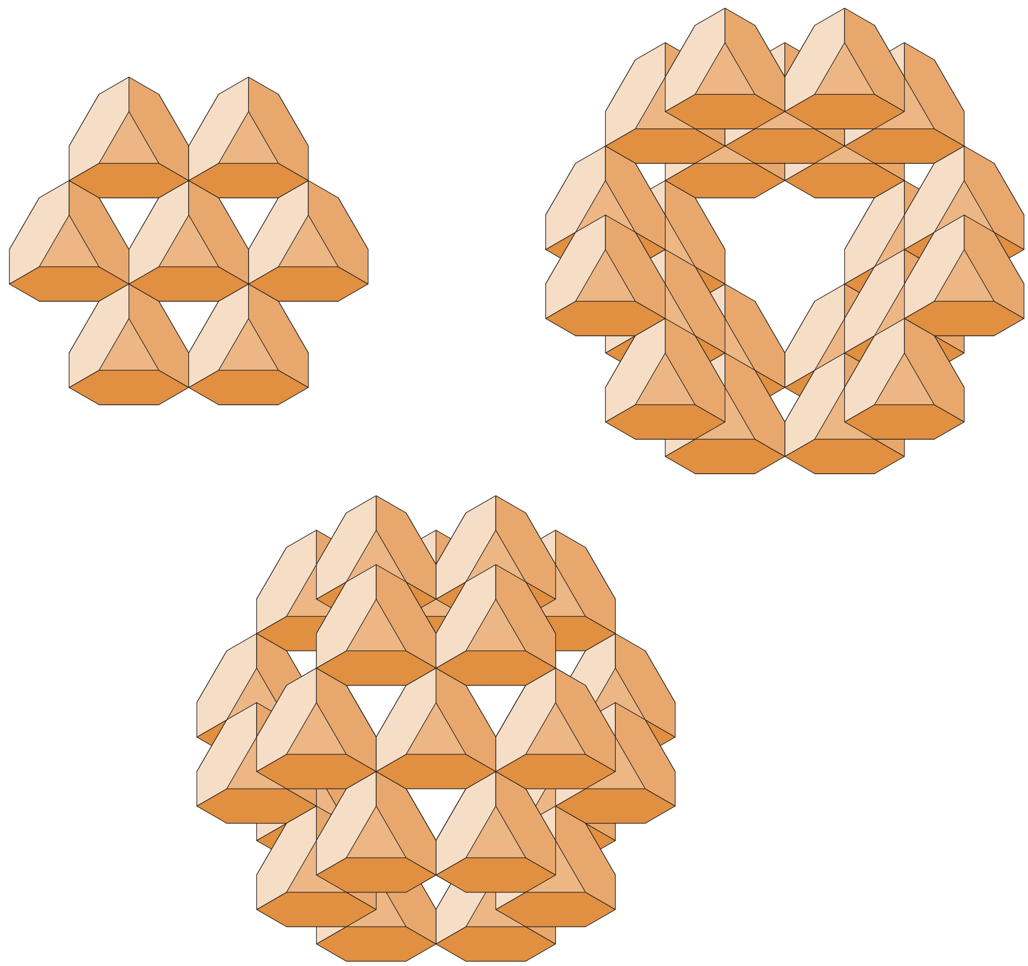

Fig. 11 A truncated tetrahedral assembly built of panels of truncated tetrahedra

An hexagonal panel can be constructed using identical truncated regular tetrahedra. Four identical panels can join edge to edge to form a truncated tetrahedron.

At top left, six truncated tetrahedra are joined edge to edge to form an hexagonal panel.

At top right, three identical panels are joined edge to edge to form three hexagonal faces of a truncated tetrahedral assembly.

At bottom, a fourth hexagonal panel completes the assembly.

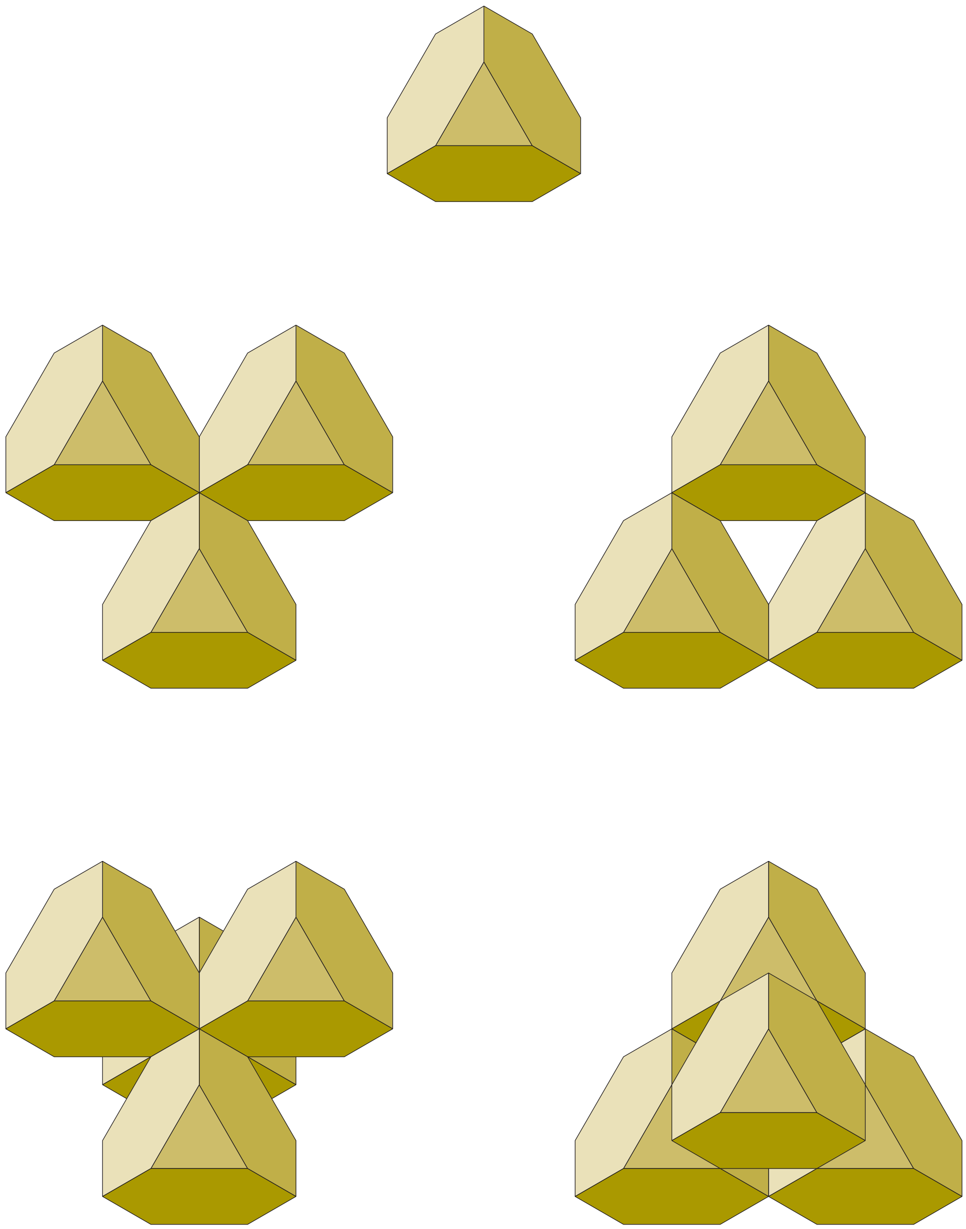

Fig. 12 Two tetrahedral assemblies of truncated tetrahedra

The figure shows two ways in which four identical truncated tetrahedra can join edge to edge to form a tetrahedral assembly. At top, the building unit stands alone.

At top left, three truncated tetrahedra are joined edge to edge to edge so that each contributes a triangular face to an open faced tetrahedron. At bottom left, a fourth unit adds the fourth triangular face to complete the tetrahedral assembly.

At top right, three units are joined edge to edge so that each contributes an hexagonal face to an open faced truncated tetrahedral assembly. At bottom right, a fourth unit adds the fourth hexagonal face to complete the assembly.

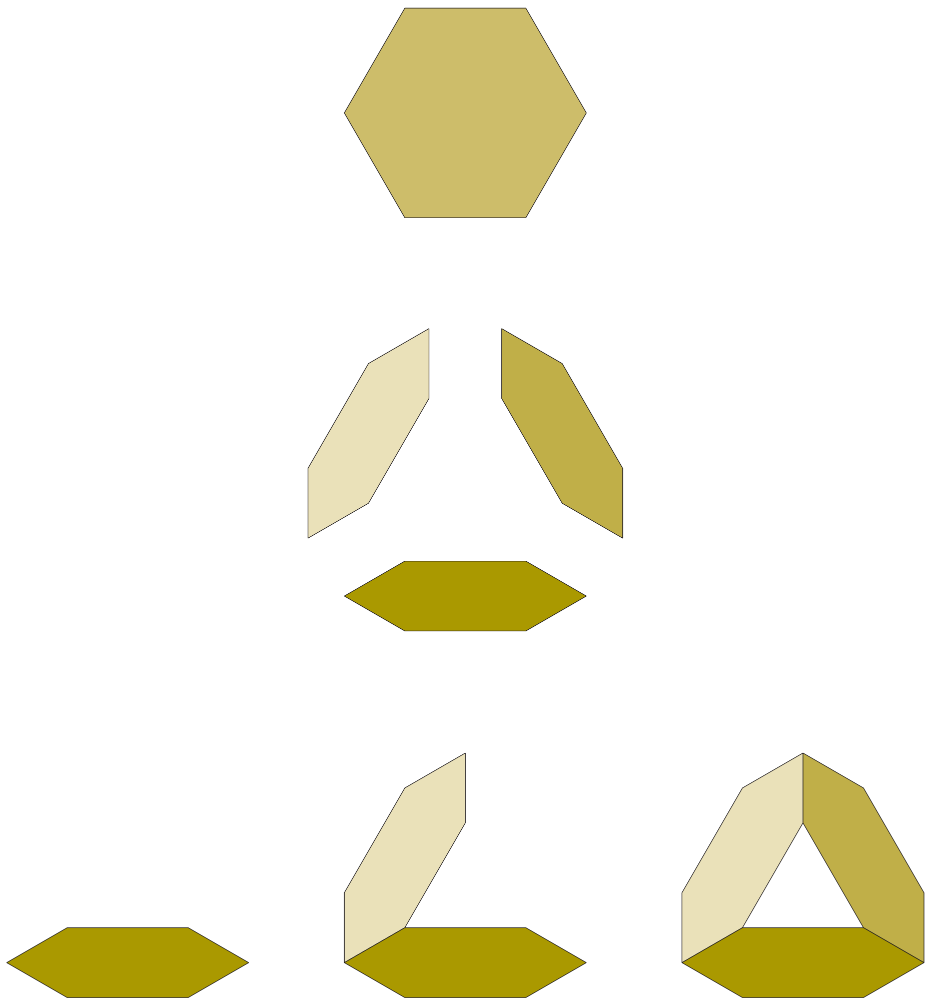

Fig. 13 Truncated tetrahedral assembly of hinge joined regular hexagonal panels
The figure shows how six identical regular hexagonal panels can be hinge-joined to form a rigid structural assembly which has the form of an open-faced truncated tetrahedron. At the top, the panel is shown in plan view. Below it, three identical panels are oriented as the remaining three faces of a regular tetrachedron. At bottom, the three panels are assembled to form an open-faced truncated tetrahedron

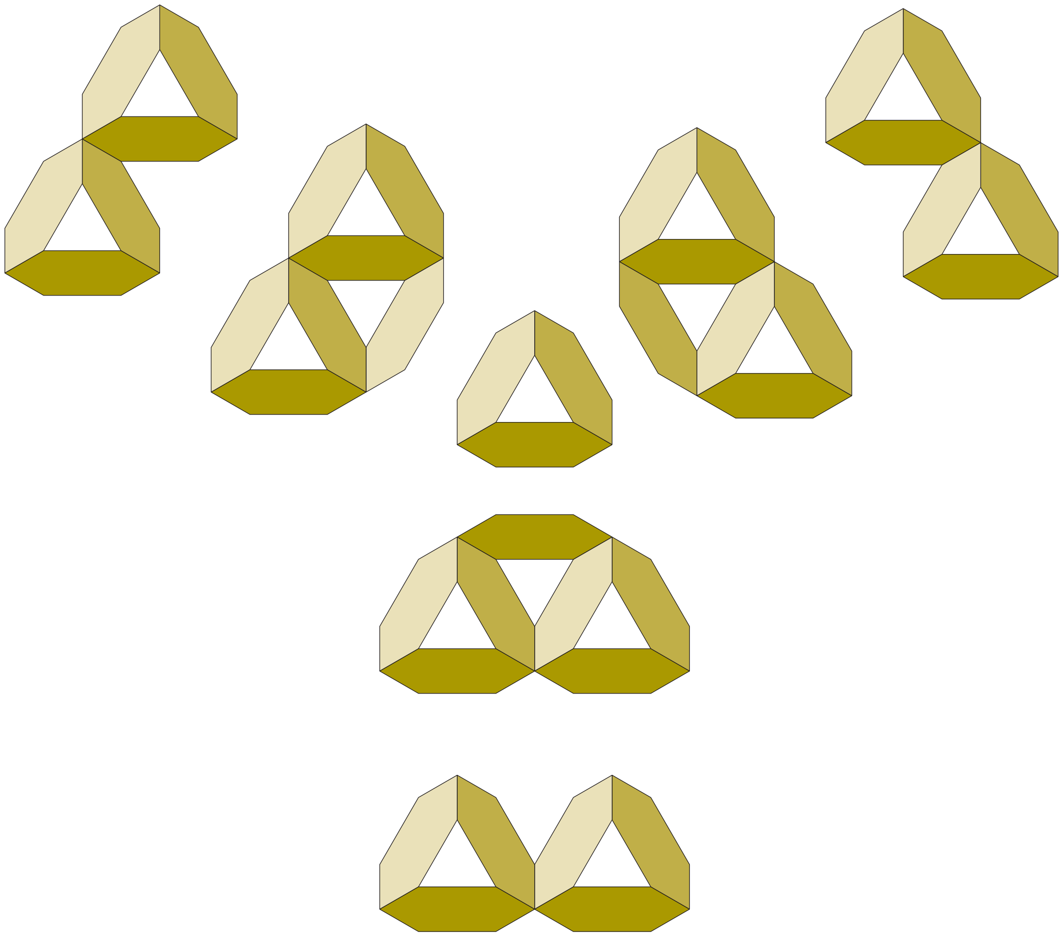

Fig. 14 Linear assembly of identical open-faced truncated tetrahedra

The figure shows how identical open-faced truncated tetrahedral assemblies consisting of hinge-joined regular hexagonal panels can be hinge-joined in linear assemblies. Each pair of joined tetrahedra is stabilized by an additional panel. This added panel combines with a panel of each tetrahedron to form an inverted tetrahedral assembly.

At center, there is a lone tetrahedral assembly. On its left, there is one pair of tetrahedral assemblies which ia hinge joined and a second pair which is both hinge joined and stabilized. On the right and below, identical arrangements of pairs are shown. The directions of the three linear pairings are parallel one of the edges of a regular triangle.

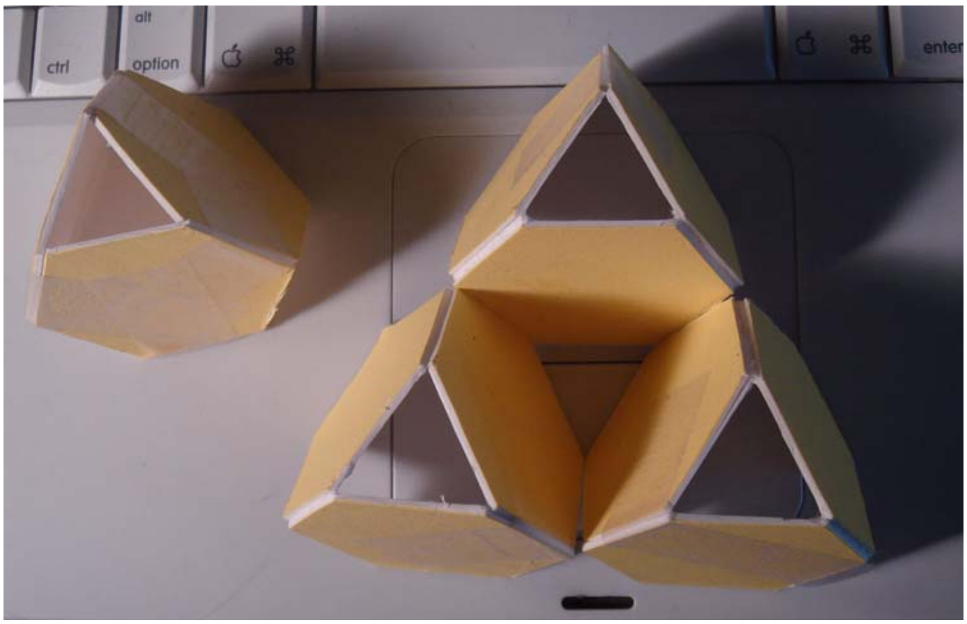

Fig. 15 Truncated tetrahedral assemblies of hexagonal panels
The photograph shows two assemblies each made of identical regular hexagonal panels joined edge to edge.

The assembly on the left is composed of four panels each providing a hexagonal face of a truncated tetrahedron.

The assembly on the right is composed of three identical assemblies of three panels each joined edge to edge. The nine panels enclose four open faced truncated tetrahedral volumes.

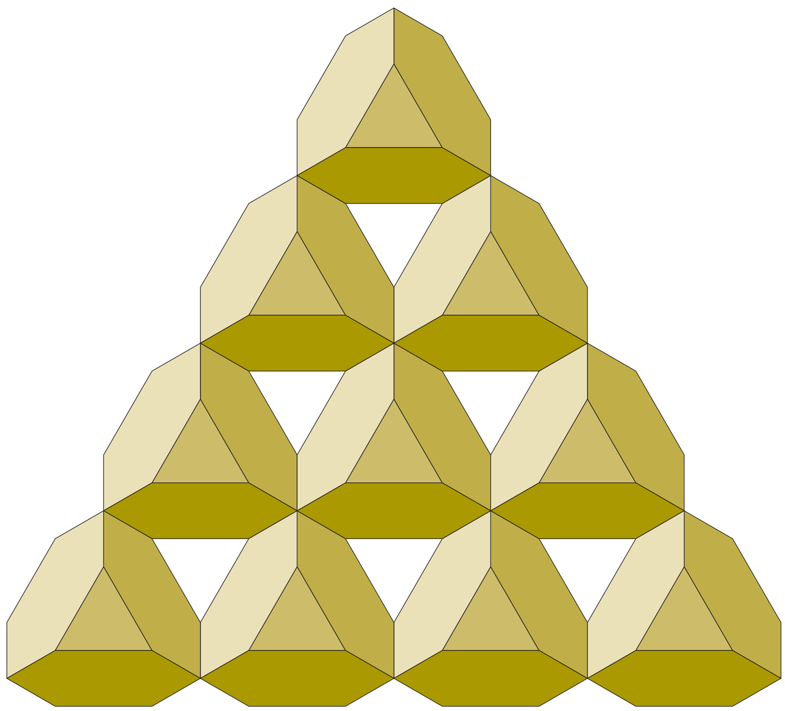

Fig. 16 Triangular panel of ten truncated tetrahedral assemblies
The figure shows a triangular panel composed of ten identical truncated tetrahedral assemblies each consisting of four edge joined regular hexagonal panels. This arrangement is similar to the triangular panel of diamond CFUs for the C_{1280} fullerene. [See Figure 3 of Reference 6.]

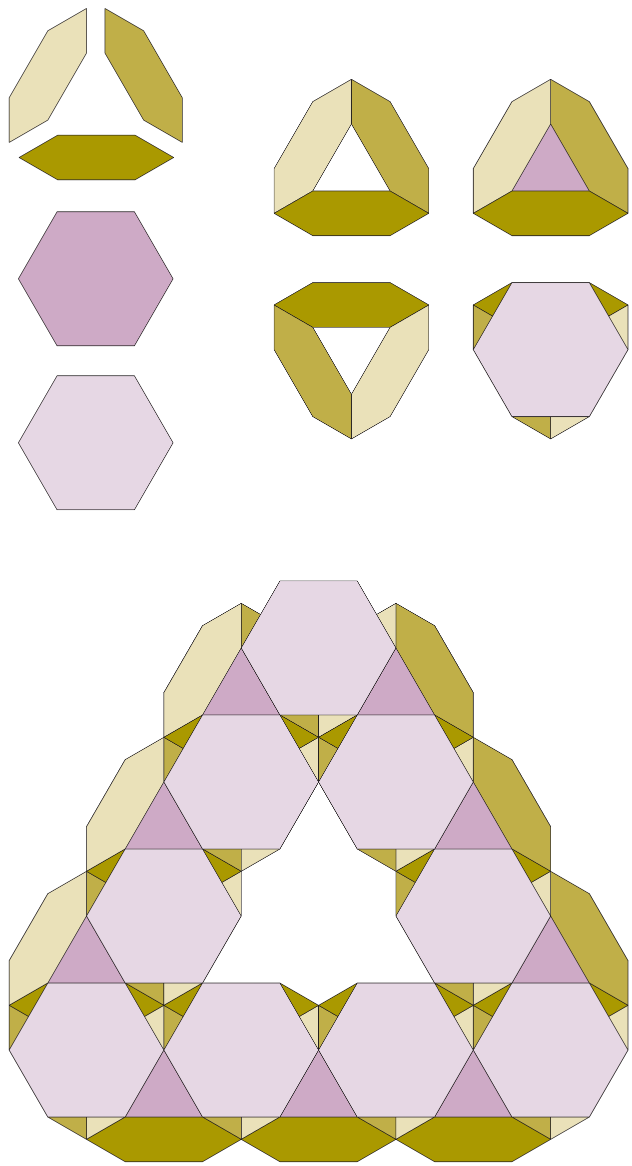

Fig. 17 Truncated triangular panel of hexagonal panels

The figure shows a truncated triangular panel composed of identical hexagonal panels that is similar to a panel of diamond CFUs which could join with identical panels to form fullerenes. [See Figure 13 of Reference 7.]

Each of the hexagonal panels is colored according to its orientation–the violet panels are parallel to the plane of the drawing; the yellow panels make an angle with the plane of the drawing. Each type of panel is shown at the top left. The relationship of each of the violet panels with the yellow panels is shown at the top right.

There are fifty-four panels in the assembly at the bottom of the figure–thirty-six yellow panels, nine dark violet panels, and nine light violet panels.