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http://homepage.mac.com/whitby/

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Reference

Octahedron1stEd.pdf–bookmark MINERAL–pages 90-104

Introduction

This material is excerpted from *Octahedron*. It shows how the orientation of the octahedral particle can be determined from the symmetry of crystalline twins.

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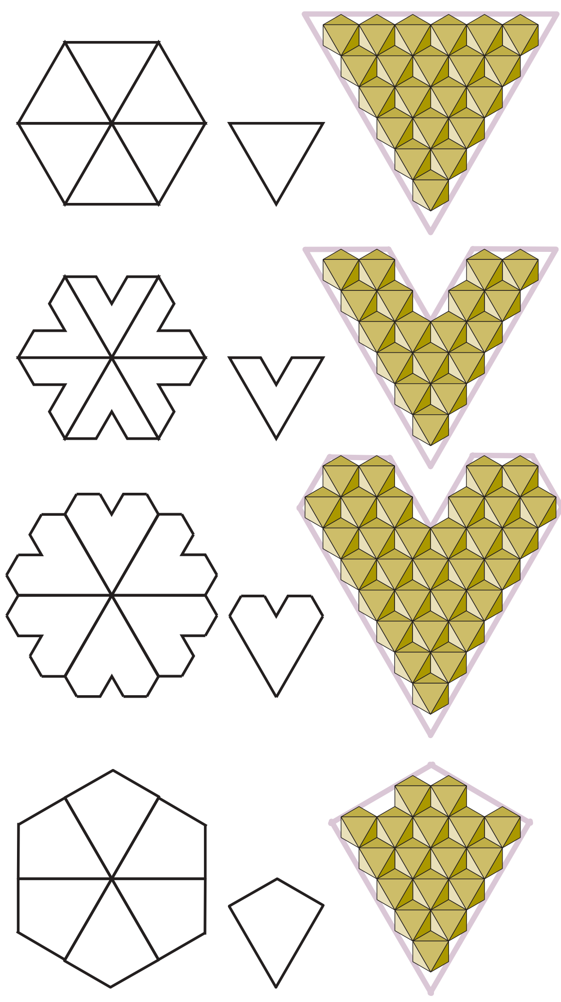
Minerals whose crystals belong to neither the isometric class nor the hexagonal class produce twinned crystals which have symmetries characteristic of those classes. The twin is termed a *pseudomorph*. The symmetry often indicates the orientation of the epn.

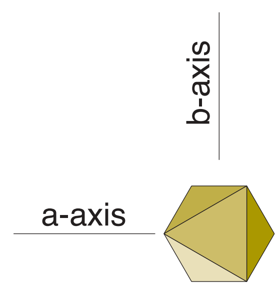
Crystals of the orthorhombic class occur in what are referred to as *hexagonal pseudomorphs*. The figure shows four forms of naturally



occurring sixlings. The left column shows an outline of the form of the sixling. The middle column shows an outline of the panel which makes up the perceived sixling. The right column show each of the panels defined by an octahedral assembly. The hexagonal nature of the sixlings requires that the octahedron of which the atoms are formed be in an orientation in which the axis of sixfold symmetry is parallel to one of its facial diameters.

Chrysoberyl

Chrysoberyl crystals are orthorhombic, having three mutually perpendicular axes of unequal lengths. Twinning results in hexagonal sixlings.[1] Because the twin is hexagonal, its axis of symmetry must lie parallel to a facial diameter of the He-octa. The incremental lengths along the threefold-axis must be an integral multiple of three times the facial diameter of the He-octa which is $s \times \sqrt{\frac{2}{3}}$, where s is the edge length of the He-octa. Of the two remaining axes, one is parallel to an edge of



Chrysoberyl axes.
View parallel to c-axis of chrysoberyl axes.

the He-octa and one is parallel to a facial altitude which is perpendicular to that edge. The relationship of the latter two axes is shown in the figure.

The axial ratios for chrysoberyl[2] are a/b=0.4701 and c/b=0.5800. In this case, the a-axis is listed as the three-fold axis. Dividing

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1. John Sinkankas *Mineralogy*, Van Nostrand Reinhold 1964, p.348
2. E. S. Dana & W. E. Ford *A Textbook of Mineralogy*, 4th ed., John Wiley 1932, p. 494

the ratio of a/b by $\sqrt{\frac{2}{3}}$ equals $1/(\sqrt{3})$ times 0.9972. The ratio of c/b equals $1/(\sqrt{3})$ times 1.0045. This latter ratio must hold for the two orthorhombic axes which are perpendicular to the threefold axis for the case of hexagonal twins. Both the axial ratios have $\sqrt{3}$ as part of their denominators so the b-axis is parallel to a facial altitude of the He-octa. The c-axis is parallel to the edge which is perpendicular to the facial altitude.

Working from the axial ratios and the requirements of the octahedral assembly, the following axial values are found

$$a = 3 \times \sqrt{\frac{2}{3}} \times s$$

$$b = 6 \times \frac{\sqrt{3}}{2} \times s$$

$$c = 6 \times \frac{s}{2}$$

Axial values for chrysoberyl are 4.42, 9.39, and 5.47. These values can be used to obtain a value for s which is an integral multiple of the edge length of the He-octa.

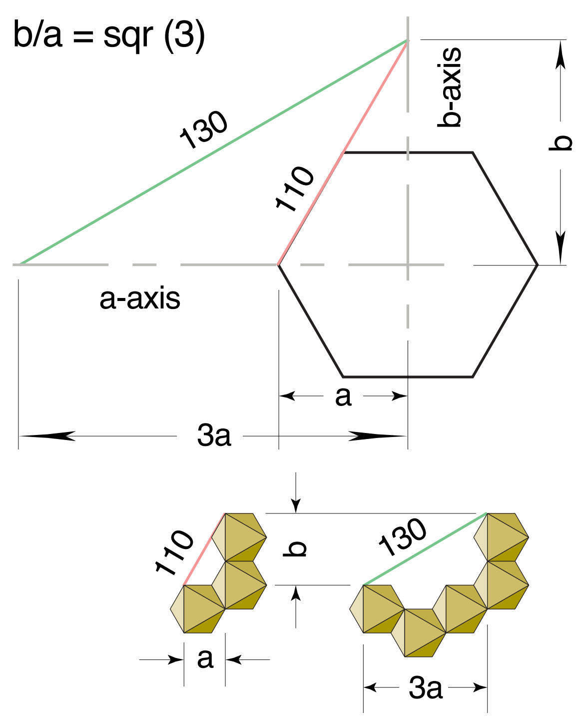
$$s = \frac{4.42}{3 \times \sqrt{\frac{2}{3}}} = 1.8045$$

$$s = \frac{9.39}{6 \times \frac{\sqrt{3}}{2}} = 1.8071$$

$$s = \frac{5.47}{3} = 1.8233$$

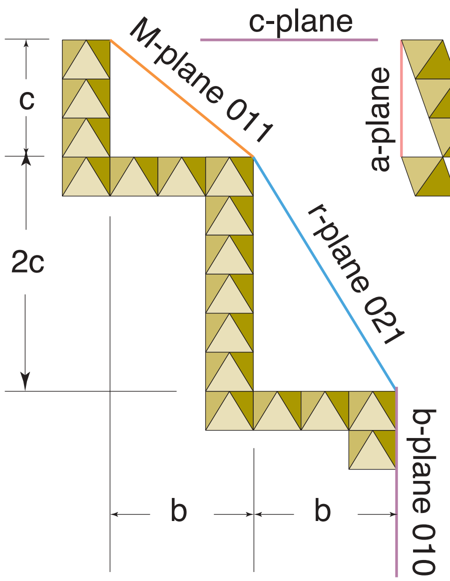
Axial relationships

For orthorhombic crystals like chrysoberyl whose twins have hexagonal symmetry, the ratio of the two axes perpendicular to the axis of symmetry is $\sqrt{3}$. These two axes are labeled a and b in the figure. The relationship of the planes which are parallel to the axis of symmetry, the 110-plane and the 130-plane, is shown in the upper part of the figure. They are shown with the two axes and all are related to the regular hexagon. The lower part of the figure shows how the planes are defined by regular octahedra. The 110-plane is shown to be defined by octahedral edges. The 130-plane is defined by octahedral vertexes and is parallel to an edge of the octahedral face which is perpendicular to the axis of symmetry. The 130-plane is the twinning plane in chrysoberyl sixlings.



Relationship of orthorhombic sixling planes to the octahedron.

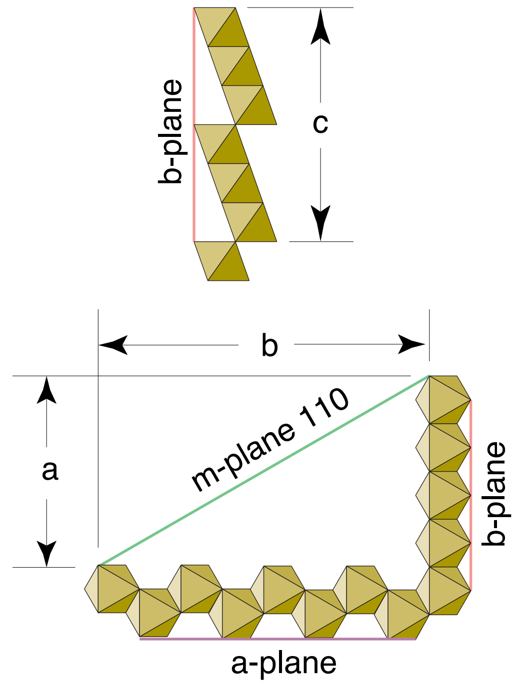
The relationship of the planes of chrysoberyl which are parallel to the a-axis (parallel to a facial altiude of the octahedron) is shown in the next figure. In the upper right, the octahedra which define the a-plane are shown. The 011-plane and the 021-plane are defined by octahedral vertexes.



Aragonite

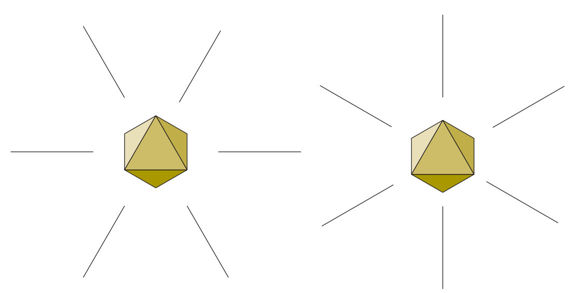
Table 12: Aragonite series axes

Mineral	Axial factors	
s, Å	a-axis	b-axis
Aragonite	1.0580	0.9822
1.1696		
Cerrusite	1.0323	0.9802
1.2471		
Strontianite	1.0316	0.9776
1.2431		
Witherite	0.9835	0.9554
1.3370		



Sixling axes.

Hexagonally symmetrical sixling axes must project onto a plane which is parallel to a face of the regular octahedron. Two of the simplest arrangement of sixling axes are shown in the figure. The axes around the octahedron in the

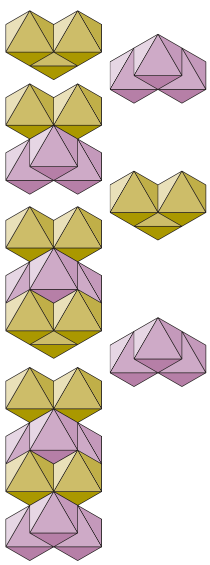


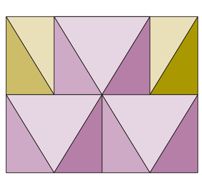
upper part of the figure are parallel to the edges of the upper face of the octahedron. Each is parallel to the upper face of the octahedron. They are called *edgial-axes*. The axes of the lower octahedron differ from those about the upper octahedron in being perpendicular to the edges of the upper face of the octahedron. They are called *special-axes*.

Special-axised sixlings.[1]

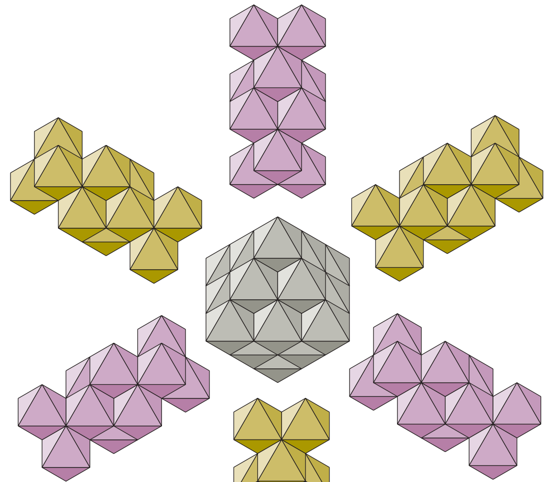
Sixlings can be viewed as spokes attached to a hub. A special-axised spoke can be assembled using octahedral triplets.The figure below shows the assembly of a spoke consisting of four triplets.

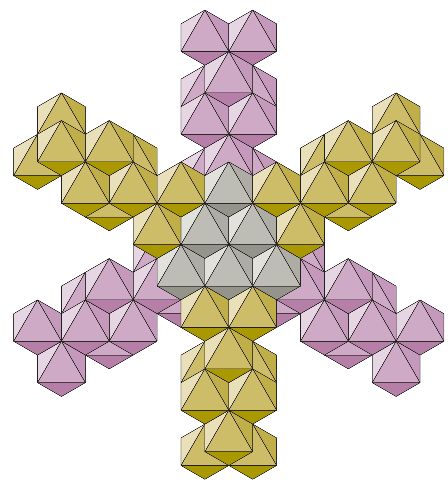
1. Dana & Ford, *ibidem* Fig. 479, p. 193





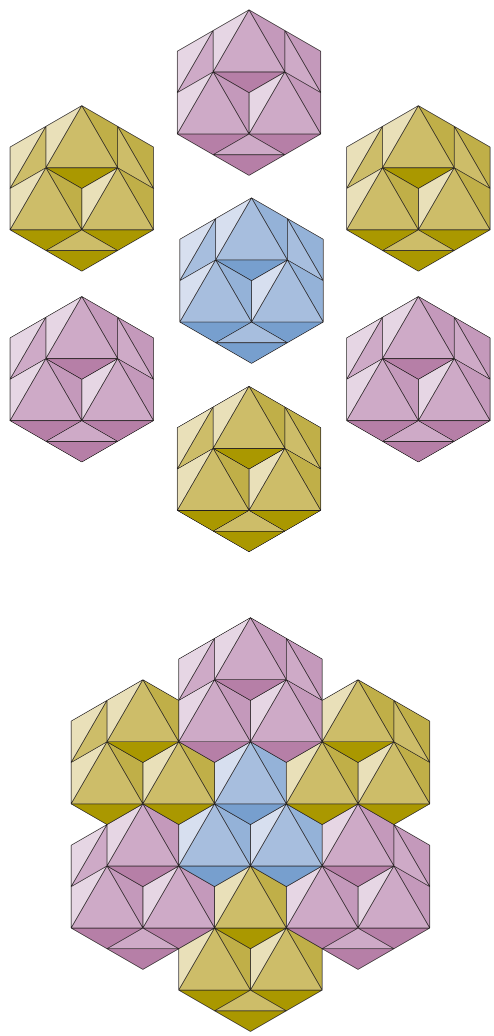
Axial view of spoke.



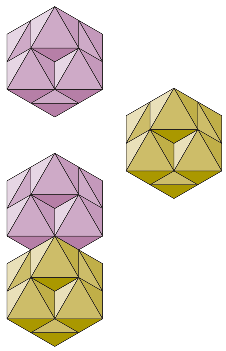


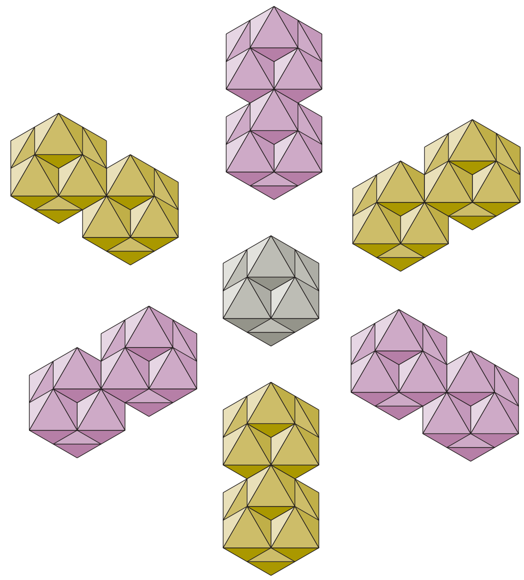
In the top figure, six spokes are arrayed around a 3-octa which is to serve as their hub. The six spokes are in two sets of three spokes each. Each spoke is rotated 120° to each of the other spokes in its set. Adjacent spokes differ by a half rotation about the spoke axis.

The spokes are joined to the hub in the bottom figure.

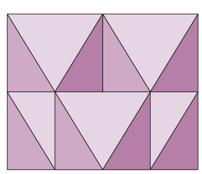


Tabular crystal with reentrant planes parallel to octahedral edges.
The axis of hexagonal symmetry is parallel to a facial diameter of the octahedron. This form is found in chrysoberyl of the orthorhombic class.A tabular crystal can be composed solely of 2-octas in the arrangement which is shown in the next figure. A 2-octa hub is shown in blue with six 2-octas arranged about it to act as the six spokes

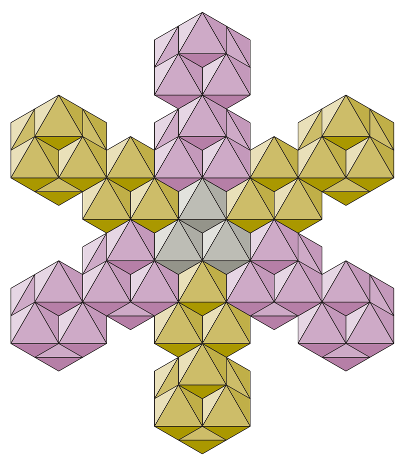




Each of the 2-octa spokes can be lengthened by adding another 2-octa in the manner shown in the figure. The radial view of the spoke is shown here.



Axial view of special axised spoke



Special axis sixling twin [1]
The hub and spokes are joined here.

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1. Dana & Ford, *ibidem*, Fig. 479, p. 193

Rutile sixling.[1]

Rutile is a tetragonal crystal which occurs in hexagonal rings. These rings can be formed in two ways. One uses specially-axised units like those shown in the figure. The ring is composed of 2-octas only. With a 1-octa mounted on each of the two faces of the 2-octas which are normal to the axis of symmetry, an octagonal array of planes about the axis of each unit is produced. The planes are suggestive of the arrangement of the planes in the rutile sixling.

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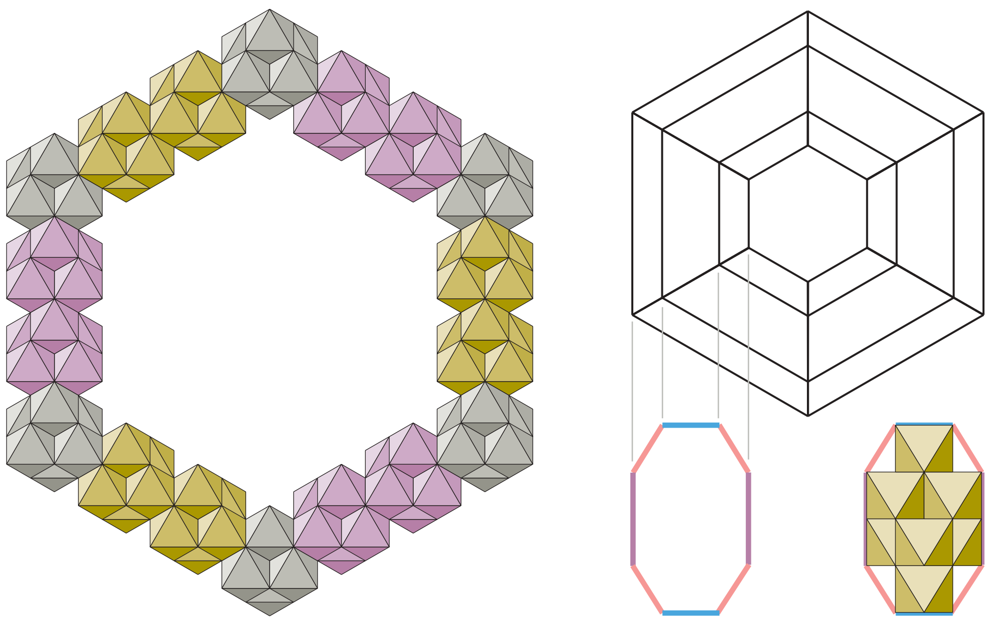
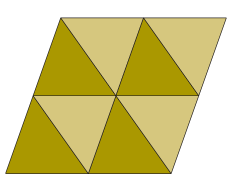
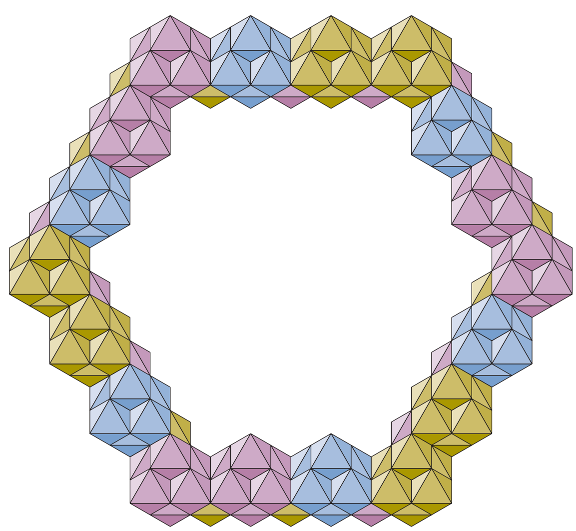
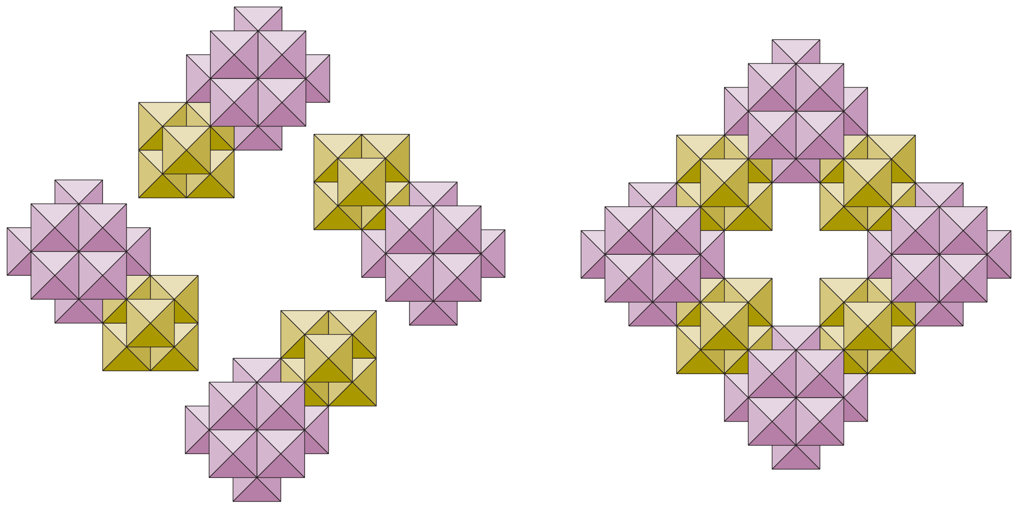
1. Sinkankas, *ibidem*, Fig. 40, p. 99



The next ring is composed of edgially-axised units. The units are composed of 2-octas with 1-octa stabilizers

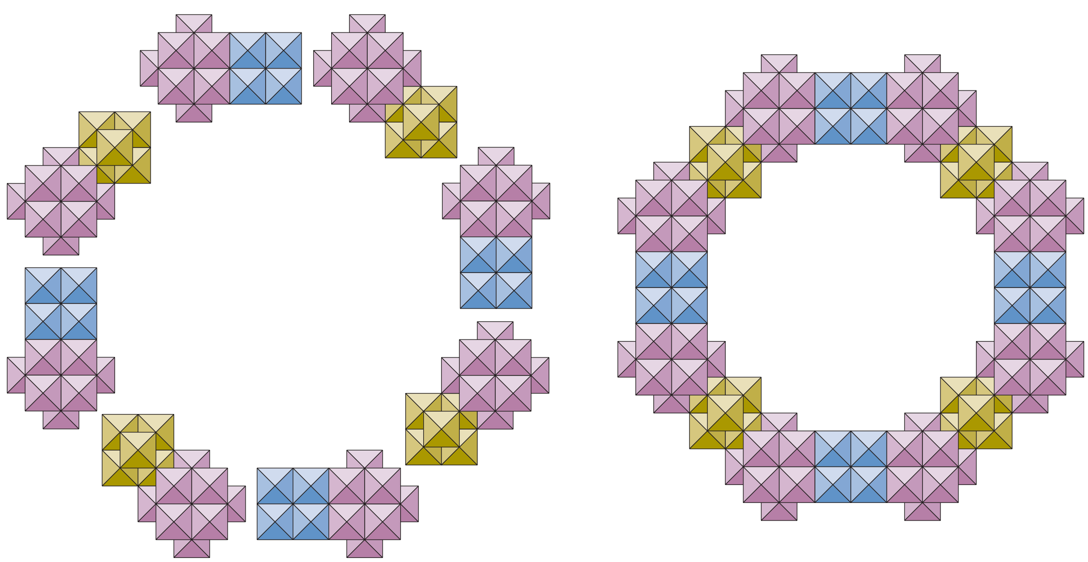






Tetragonal ring

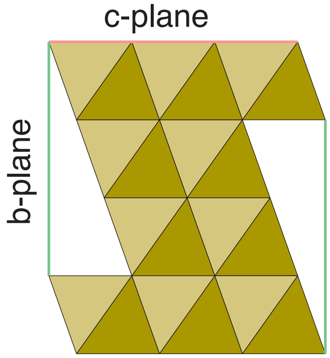
A fourling with vertexial octahedral axes is depicted here. Each leg is composed of two units—a 3-octa absent its vertexial octas and a 2-octa. An octa of the 2-octa replaces an absent octa of the 3-octa in forming the unit. The four units which are to form the ring assembly are shown in the left figure. The units join so that an octa of the 2-octa of each unit substitutes for an absent octa of the 3-octa of another unit to form the assembly on the right.



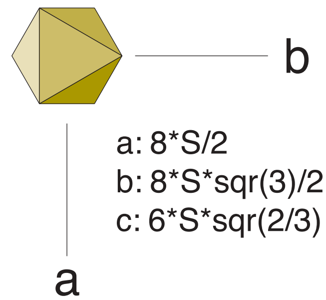
Octagonal ring

An eightling which is truly a double fourling is built from the eight units shown in the left figure. The four units of the previous ring are combined with four new units. Each new unit is composed of a 3-octa from which the vertexial octas have been removed and a second 3-octa from which the vertexial octas and four edgial octas have been removed. The latter is shown in blue. An edgial octa of the violet 3-octa occupies the position of the absent edgial octa of the blue 3-octa. Each violet-yellow-violet leg is vertexially axised; each violet-blue-violet leg is edgially axised. The completed ring is shown on the right. The yellow 2-octas fill vertexial voids in the violet 3-octas and the violet 3-octas fill edgial voids in the blue 3-octas.

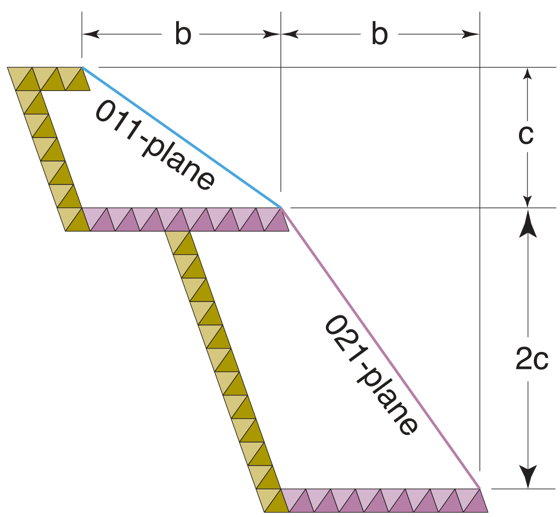
Cerrusite



Cerrusite planes
The figure shows the arrangement of the b-plane and the c-plane of the cerrusite crystal relative to the octahedron. The view is parallel to the a-axis.

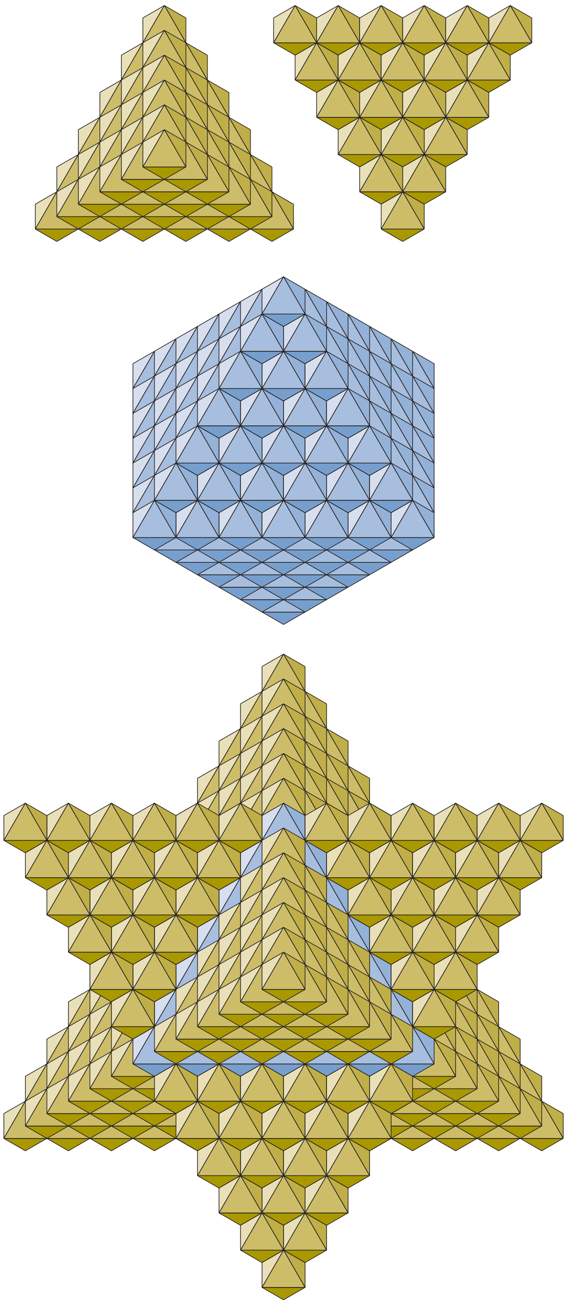


Cerrusite axes.
The axes of the twins should not be confused with the hexagonal axes. Each of the spokes is considered a separate crystal of the orthorhombic class.



Cerrusite planes defined by octahedral edges.
The 011-plane and 021-plane of cerrusite are defined by octahedral edges. The two planes are shown edge-on in the figure. The view is parallel to the a-axis.

Tetrahedrite

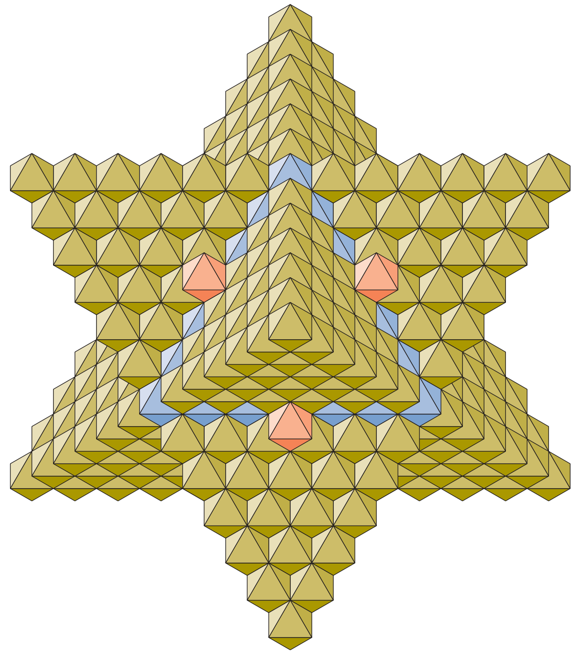


Tetrahedrite penetration twin [1]

This twin is producible by mounting a 6-tetra on each face of a 7-octa to produce the assembly at the bottom. The 7-octa is shown in blue and the 6-tetras are shown in yellow. The 6-tetras are identical and are in one of the two orientations shown at the top.

1. Dana & Ford, *ibidem*, Fig.418, p. 184

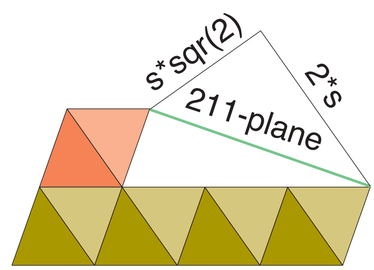
Eulytite



Eulytite penetration twin [1]

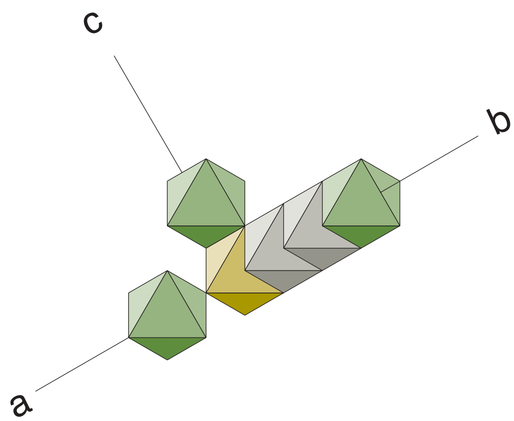
The assembly shown above is the same as that of the tetrahedrite twin. It differs by the addition a red colored octa at the middle of each of the 7-octa's edges to produce the 211-planes in the manner shown below.

1. Dana & Ford, *ibidem*, Fig. 435, p. 187



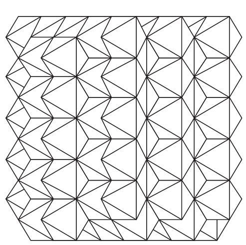
Staurolite

The axes of Staurolite are as $\sqrt{2}$:3:1. The $\sqrt{2}$ in the value for the first axis suggests means that the axis is parallel to a vertexial diameter. The other two values are integers and so these axes are each parallel to an edge which is perpendicular to the vertexial axis.



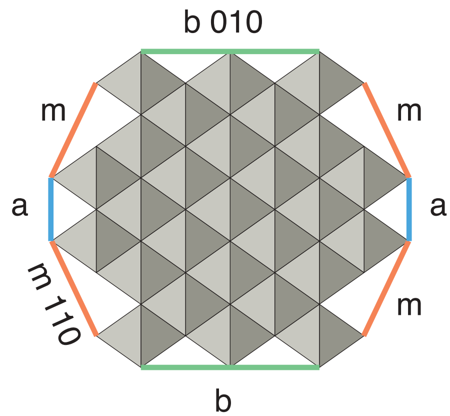
The figure shows three green colored octahedra whose centroids are on three mutually perpendicular axes. The centroid of the yellow octahedron is at the junction of the three axes. The positions of the green octahedra are as the crystalline axial ratios of staurolite.

A view of an octahedral assembly which is perpendicular to the c-axis is shown in the figure.Two c-planes are defined by the octahedral



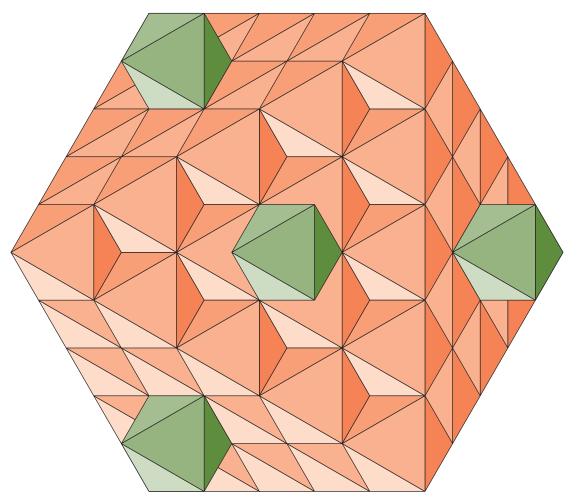
edges at the top and bottom of the assembly. Three rows of octahedra which are nearest on the right side of the assembly define a b-plane with their outermost edges.

The relationship of the planes parallel to the c-axis are seen in the next view of the same



octahedral assembly which is normal to the c-planes.

This assembly can be joined to a compound octahedron so that its centroid and the com-



pound octahedral centroid lie on the same edgial diameter. By adding additional assemblies which are identical to the first as spokes to the octahedron as hub the staurolite twins can be modeled.

Interpenetrant twins

Staurolite forms interpenetrant twins that appear as crosses. These twins are either at 90° or at 60°.

The staurolite crystal faces can be precisely modeled using the regular octahedron as the modeling unit. The *c*-axis is edgial. The *c*-planes and the *b*-planes are edgial; the *m*-planes and the *r*-planes are vertexial. An axial view of the crystal as formed by a minimal number of octahedra shows the *r*-plane defined by a vertex of each of a trio of octas at elevation #5 and a vertex of a single octa at elevation #1. This provides a *c*-axial separation between the elevational contacts of twice the edge length of the octahedral unit, or $2 \times s$. The radial separation is $\sqrt{2} \times s$. The angle of the *r*-plane with the c-axis is $\operatorname{atan}\langle \sqrt{2} \times \frac{s}{2} \times s \rangle$ which is $\operatorname{atan}\frac{1}{\sqrt{2}}$, or 35.264389699°. The angle between *r* and *c* is then 90° + $\operatorname{atan}\frac{1}{\sqrt{2}}$ or 125°, the value given in Sinkankas.

The *m*-plane is seen in the same axial view as an edge contacting the vertex of an octa at elevation #1 and an octa at elevation #2. Each contact octa is just the topmost of a column of identical octas paralleling the c axis of the crystal. The offset parallel to the a axis is one half of the vertexial diameter of the octa or $\frac{s}{\sqrt{2}}$; and that parallel to the *b*-axis is one and a half edge lengths or $3 \times \frac{s}{2}$. The *m*-plane makes an angle with the *b*-axis whose tangent is $\frac{s}{\sqrt{2}} \div \frac{3 \times s}{2}$ or the $\operatorname{atan}\frac{\sqrt{2}}{3}$ which is 25.23940182°. This gives an angle between planes *b* and *m* of 115.2394018°, and between two m planes an angle of 129.5211964°. The values from Sinkankas are 115° and 129.5°.

The 90° cross is viewable in the vertexial direction that is perpendicular to the *c*-axis of each of the twins. The 60° cross is viewable in the facial direction that is perpendicular to the *c*-axis of the each of the twins.

It becomes apparent in the modeling that the octas of the common intersection of the legs of the “individuals” belong as much to one as to the other. They are jointly held. The individuals are then mere branches of a single crystal.

Table 13: Crystal plane angle[a] comparison

Planes	Miller	Octa	Dana[b]	Octa/Dana
mm′	110^110	50°28′44″	50°40′	0.996294
cr	001^101	54°44′08″	55°16′	0.990390
rr′	101^101	109°28′16″	110°32′	0.990390
mr	110^101		42°02′	

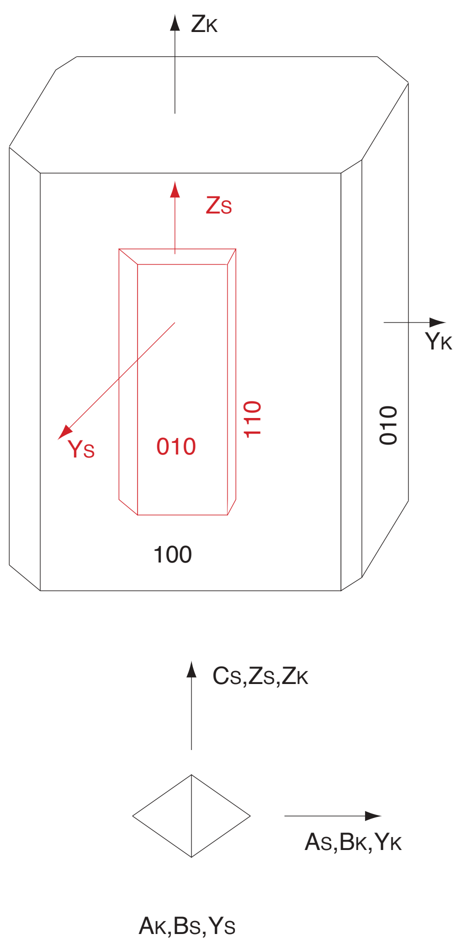
a. These are the angles between the normals to the planes and are the supplements of the interfacial angles.
b. Dana & Ford, *ibidem*, p. 637

The regular octahedron has twelve edges. These edges can be viewed as six pairs of parallel edges. These edge pairs can be viewed as paired so that together they constitute the perimeter of one of the orthographically projected vertexial views of the regular octahedron of which they are topological features. There are three such pairings.

A staurolite twin has its *c*-axis parallel to one of the edge pairs. If that edge pair makes a square with the edge pair that is parallel to the c axis of its twin, then the axes of the twins make an angle of 90°. If if does not, then the axes of the twins make an angle of 60°. Thus, of the six pairs, one pair is take up by each twin. For a given crystal, there is but one 90°

crossing. There are four 60° crossings. For each crossing there are two legs.

When the axis of a crystal is parallel to the edges of its octahedral components, then it is an edgial axis. An edgial axis viewed axially is an edgial view. The *c*-axis of a staurolite crystal is thus an edgial axis. So is the *b*-axis. But the *a*-axis is vertexial.



Staurolite crystal inside Kyanite crystal.[1]

The *b*-axis of kyanite is oriented the same as the a axis of staurolite. Assuming that the octahedra of the two crystals are in the same orientation, and referring to the orientation of the staurolite octahedra derived from the axial ratios, *b* staurolite and *a* kyanite are edgial axes.

The orientation of the epn is shown below the crystal diagram.

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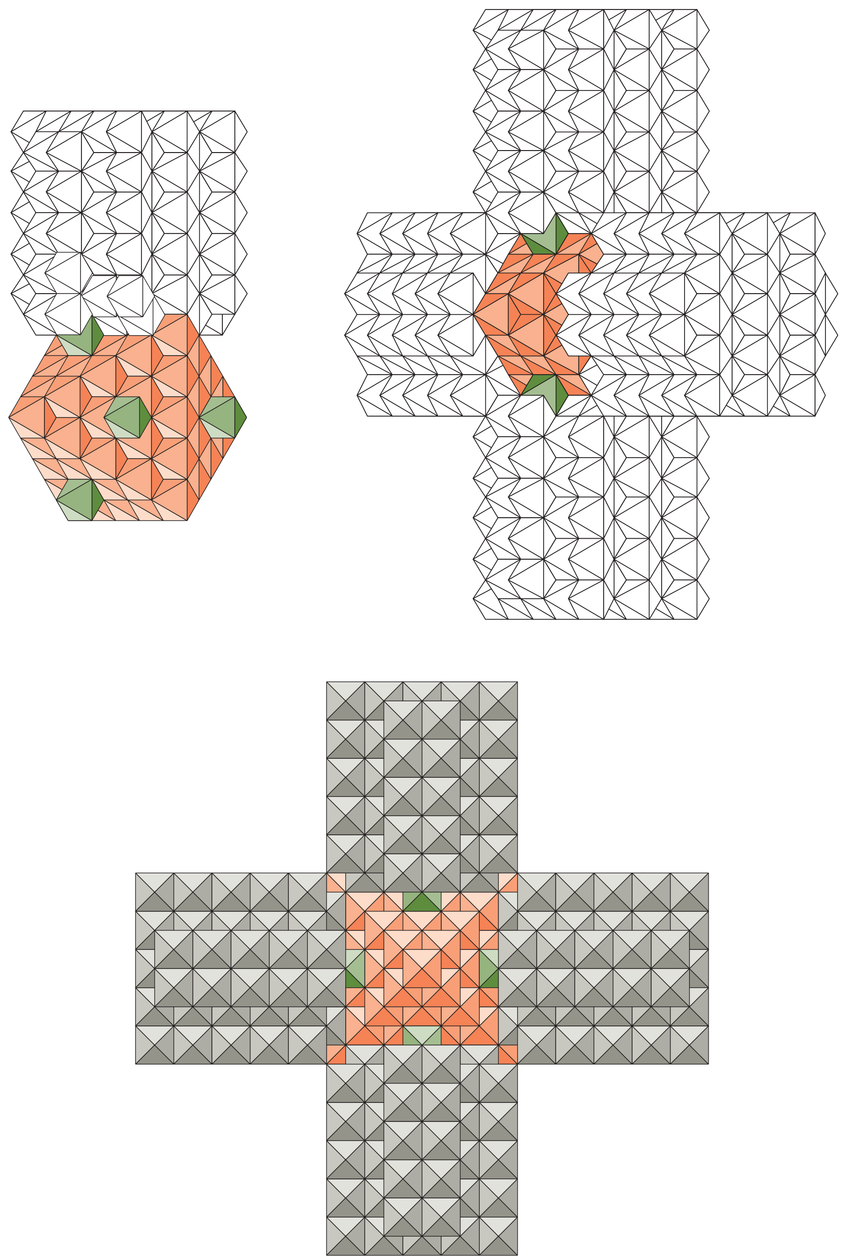
1. W. A. Deer *ibidem* Fig. 35, p. 157



Staurolite twin, right angled cross[1]

The figure on the upper left shows one spoke joined with the hub. The upper right shows the hub with four identical spokes. The same four spoke assembly is shown at the bottom wherein the axis of each spoke is parallel to the plane of the paper. The fourfold symmetry shows the right angle relationship of the spoke with each of its two neighbors.

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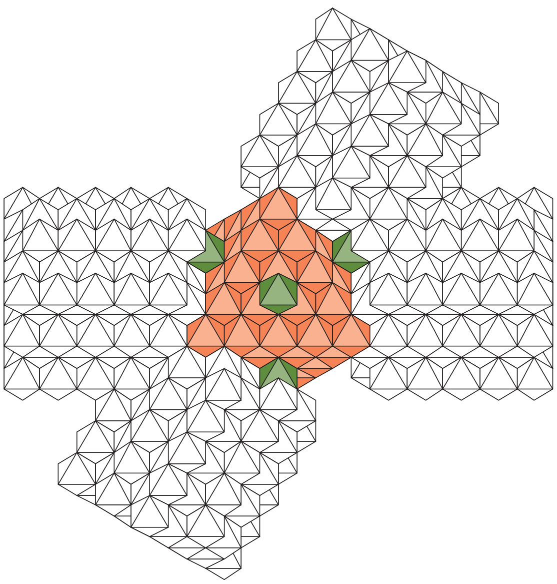
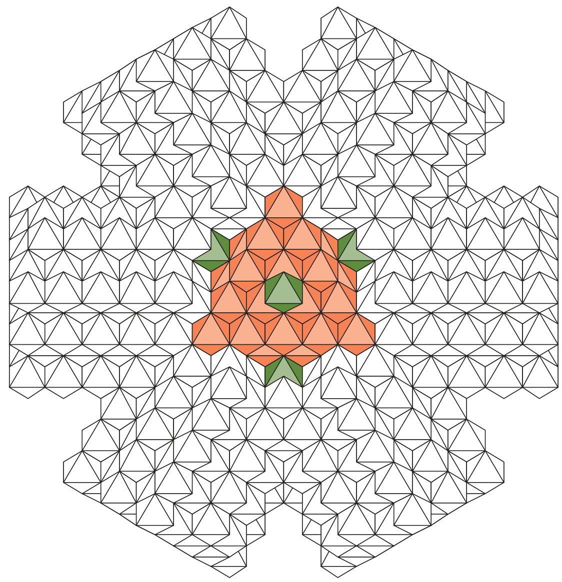
1. Dana & Ford, *ibidem*, Fig. 464, p. 191



Staurolite "sawhorse twin"[1]

The figure shows the same hub and the same four spokes of the right angle cross joined to form a 60-degree cross which is called a "saw-horse twin".

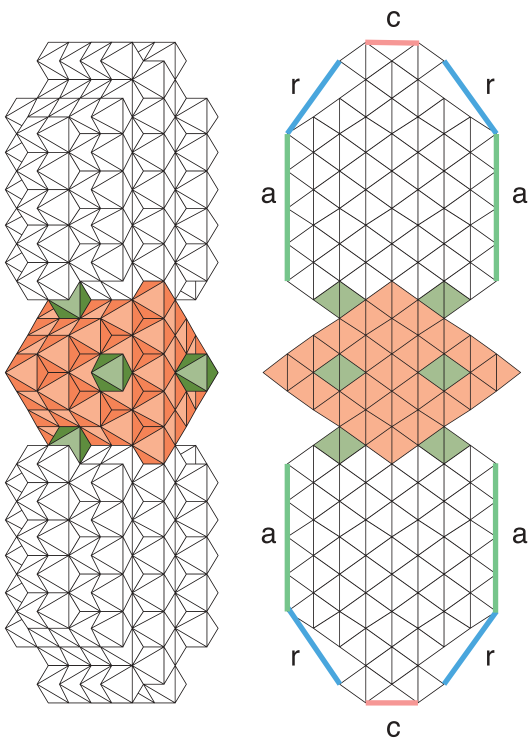
1. Dana & Ford, *ibidem*, Fig. 963, p. 638



Staurolite twin [1]

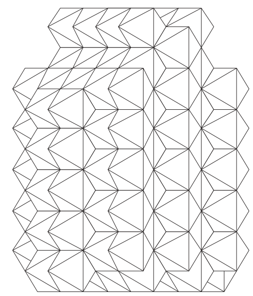
An additional pair of spokes has been added here to produce a threeway crossing which occurs in nature.

1. Dana & Ford, *ibidem*, Fig. 423, p. 185



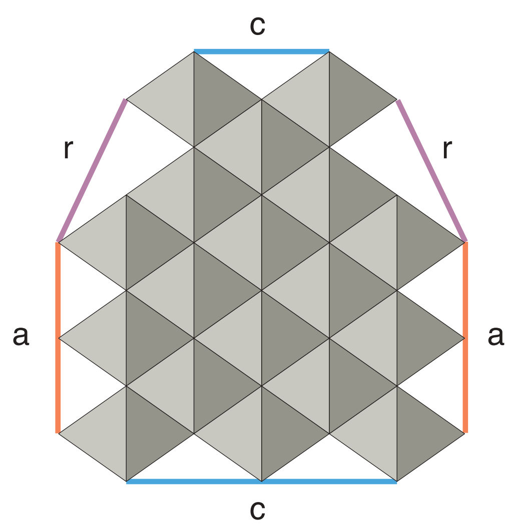
Staurolite twin with r-faces[1]

1. Dana & Ford, *ibidem*, Fig. 961, p. 638

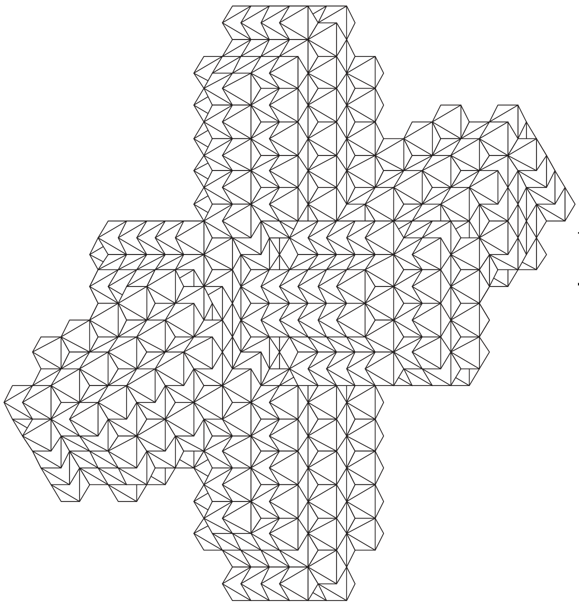


Staurolite spoke with r faces, oblique view

Prism faces *b* and *m* with *c* face on bottom and *r* and *c* faces at top.



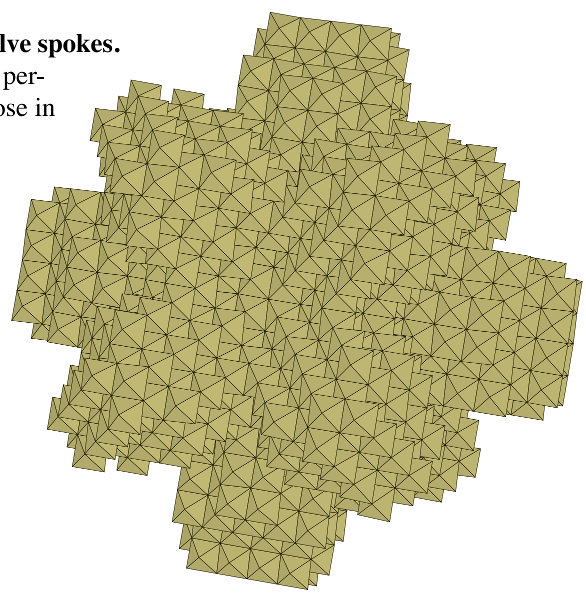
Staurolite spoke with r-faces, b-axial view

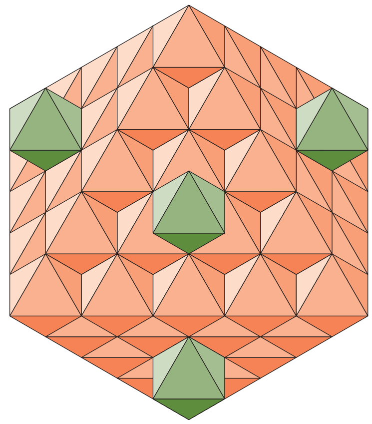


Staurolite twin combining right angle and sawhorse spokes[1].

. Dana & Ford, *ibidem*, Fig. 467, p. 191

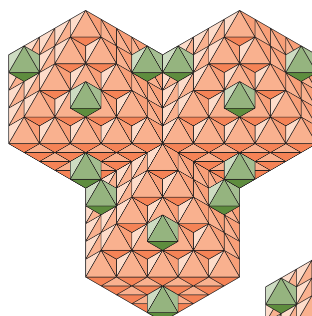
Staurolite stellar sixling having twelve spokes. This twelve spoked twin is viewed in perspective. The spokes are the same as those in the figure on the left.

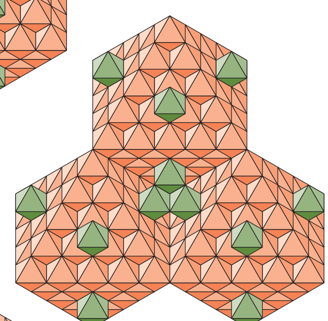


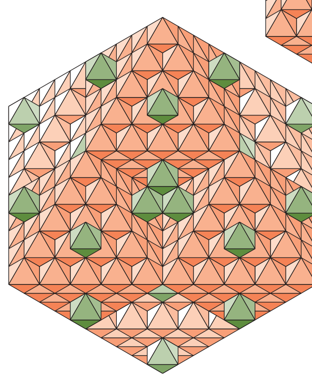


Phillipsite[1]

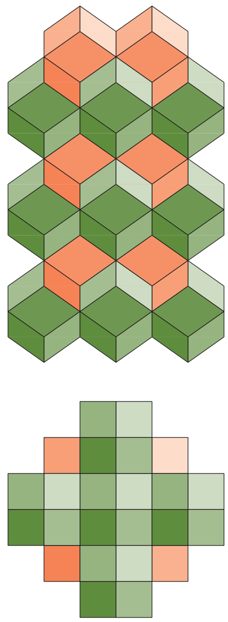
Despite its assignment to the monoclinic class, Phillipsite occurs in a number of symmetrical forms: cruciform fourling twins and rhombic dodecahedral twins. The latter can be built of identical regular octahedra arranged so that they form rhombic dodecahedral subunits and six of these subunits in crystalline association with their centroids in positional relationship as the vertexes of a regular octahedron. And so the position of the epn is established here by the symmetrical requirements of the twins.For phillipsite, the hub unit of the staurolite model becomes the modeling unit for the rhombic dodecahedral twin. The hub unit has the rhombohedral planes while including the fewest octahedra. It is a 5-octa with a 1-octa on each face at the facial centroidal position. The rhombic dodecahedral planes are defined by an edge of the 5-octa and an edge of the 1-octa on each of the faces which define the 5-octa edge.







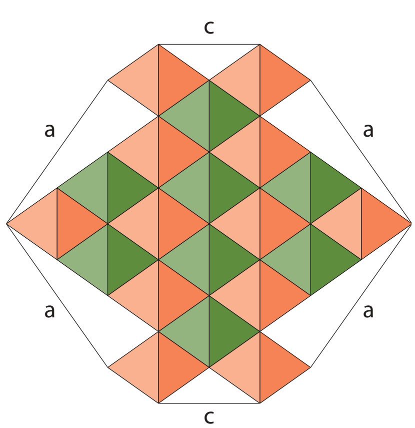
1. Dana & Ford, *ibidem*, Fig. 480, p. 193 & Fig. 426. p. 185

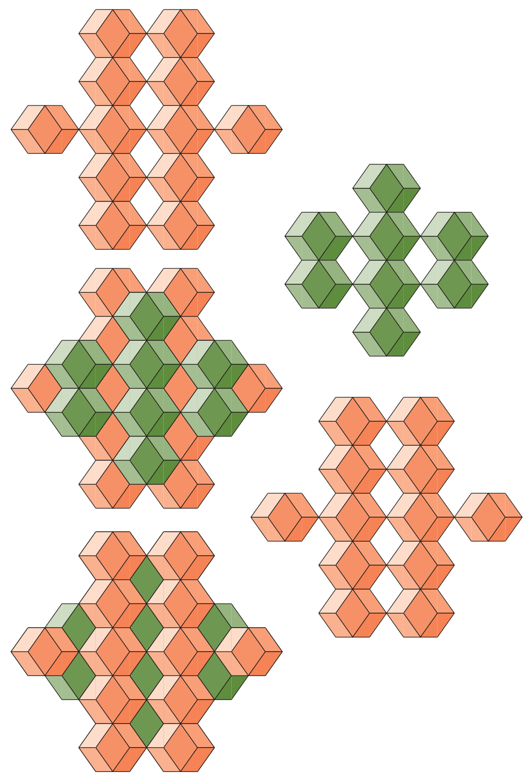


Phillipsite, cross-shaped spoke[1]

Prism faces are defined by rhombic dodecahedral faces (octahedral edges); axial (*bottom*) and radial (*top*) views

1. Dana & Ford, *ibidem*, Fig. 479, p. 193

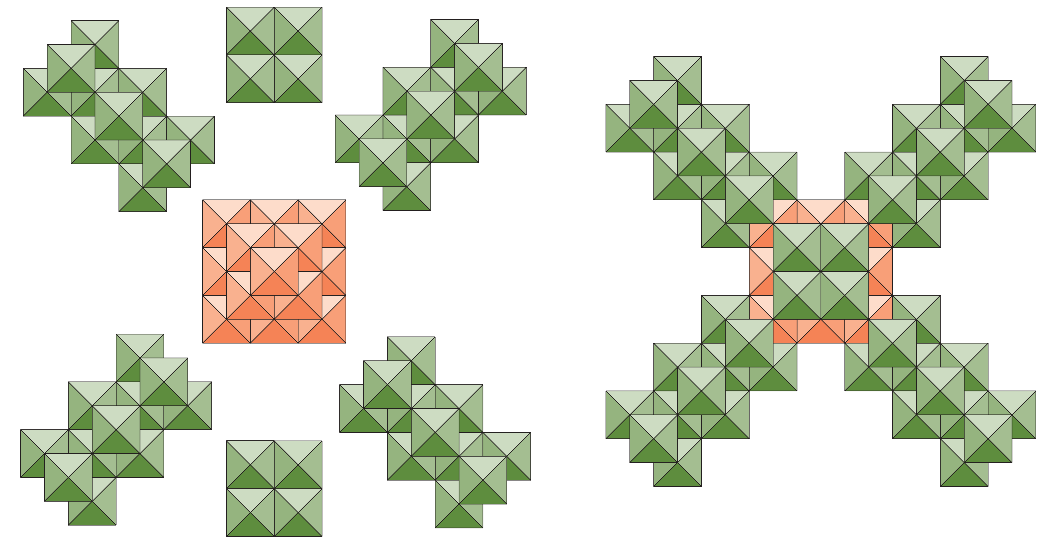




Phillipsite: b-axial view of cruciform fourling[1]

The above figure shows the assembly of the twin as minimally formed by rhombic dodecahedra. The assembly grows on the left through the addition of the layer shown on the right. The figure below left shows the twin minimally defined by octahedra.

1. Dana & Ford, *ibidem*, Fig.478, p. 193



Phillipsite twin[1]

Two views of the same phillipsite twin are depicted here. The spokes are square in cross-section and are vertexially axised. The prisms are defined by octahedral edges. The views above are octahedrally vertexial; the views below are octahedrally facial. The figures on the left are of the spokes and hubs prior to assembly. The assemblies are on the right.

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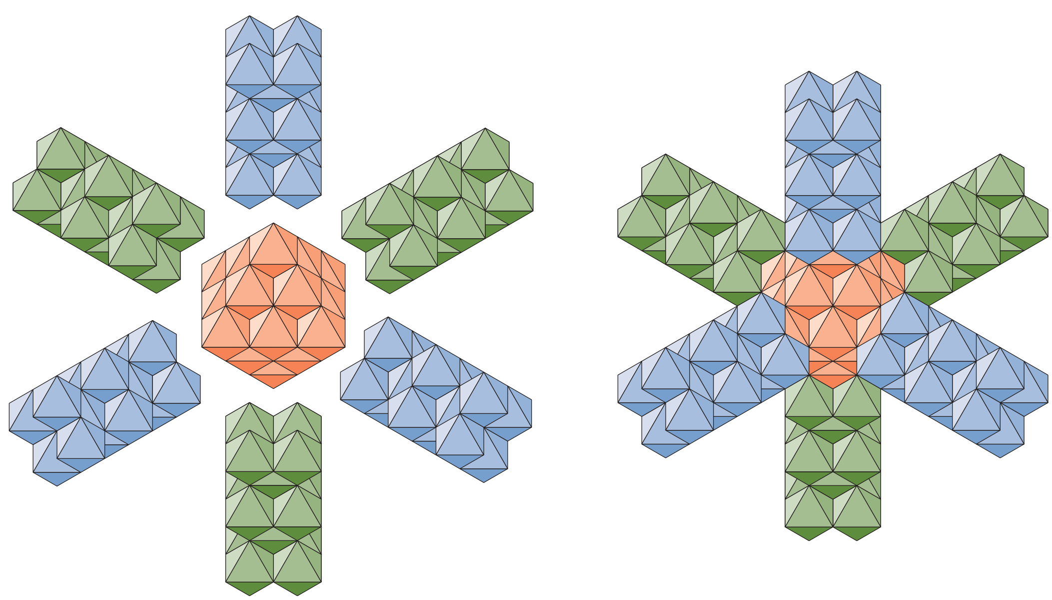
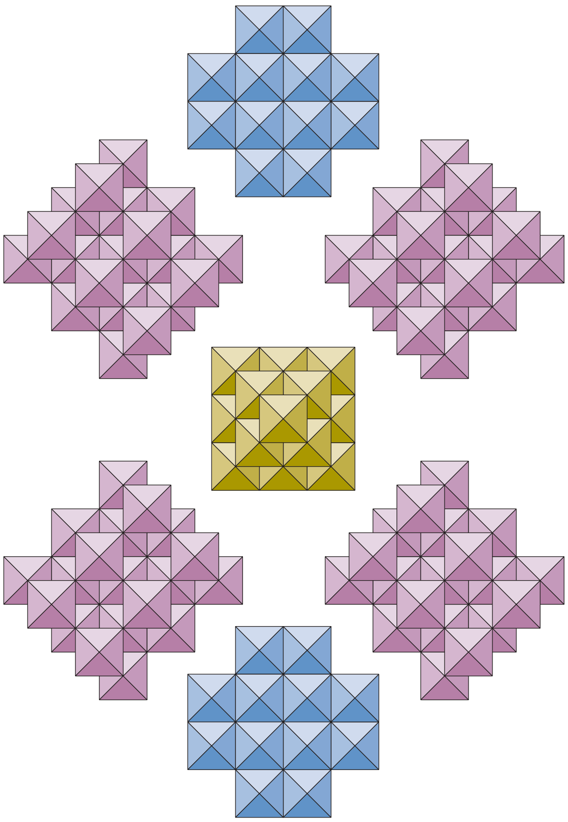
1. Dana & Ford, *ibidem*, Fig. 972, p. 647

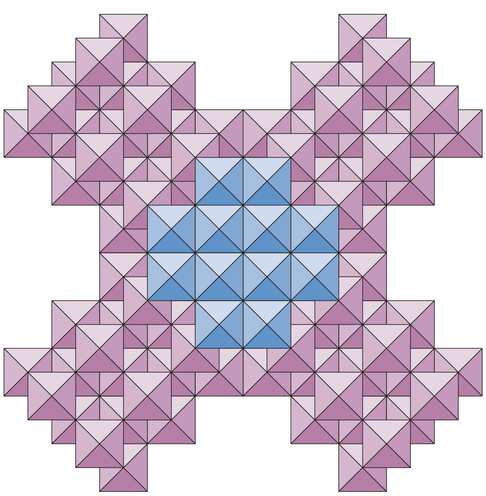


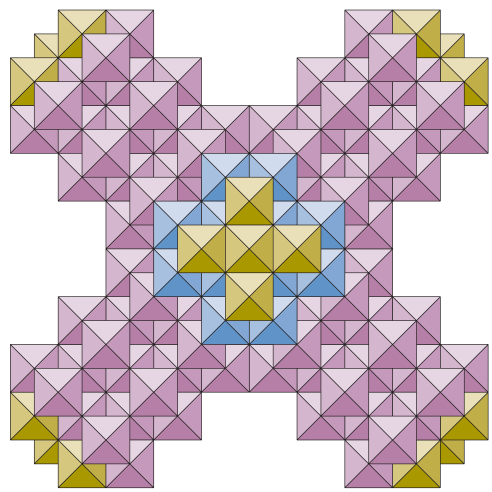
Three-way cruciform penetration twin in phillipsite [1]

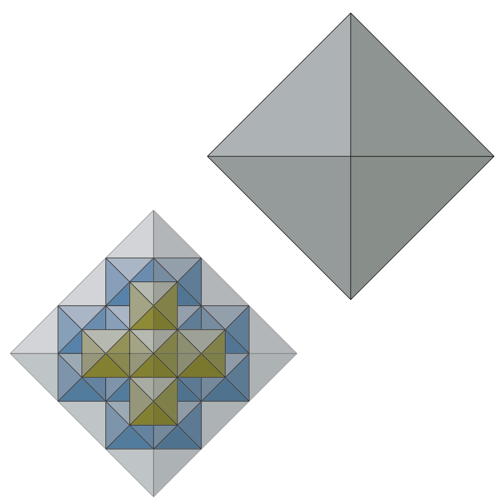
The six spokes surround the hub in the figure on the top left. The assembly is shown on the bottom left.

On the top right, the assembly has had five octahedra added to the terminus of each of the six spokes to define the pyramidal faces observed in some crystal specimens. The figure on the bottom right shows the pyramid each of whose four faces is defined by two edges of the yellow colored octas and two edges of the blue octas of the spoke.

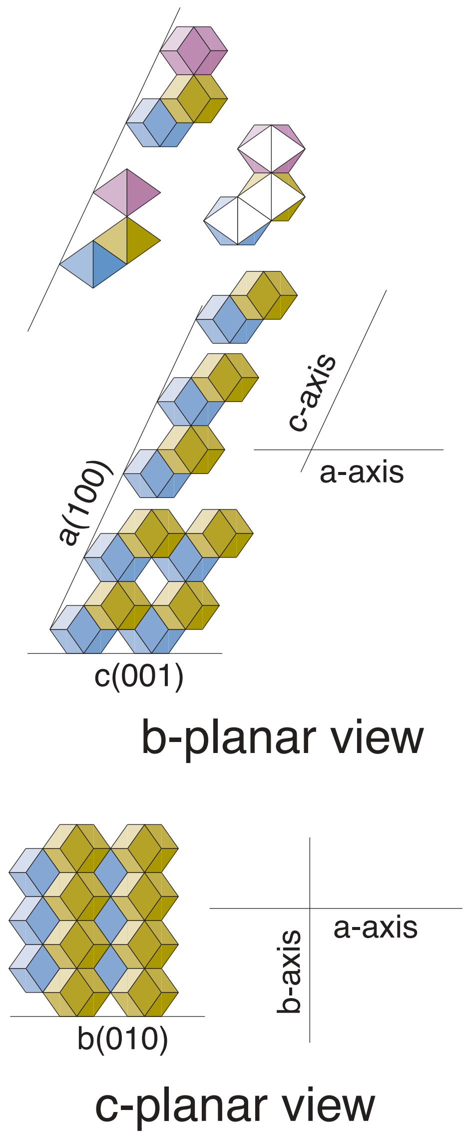








1. Dana & Ford, *ibidem*, Fig. 479, p. 193



Crystalline faces of phillipsite.

The figure shows the manner in which the crystalline faces of phillipsite are defined by rhombic dodecahedra. There are two viewing directions here. At the top the view is perpendicular to the b-plane. At the bottom, the view is perpendicular to the c-plane.

The relationship of the a-plane to the octahedron is shown at the top. This is compared with the relationship of the a-plane to the rhombic dodecahedron, with the octahedral group superimposed upon the dodecahedral group.

Two pairs of facially-joined dodecahedra are joined so as to produce the a-plane. They are joined by a like pairing to produce a c-plane.

The a-plane is defined by octahedral vertexes while the b-plane and the c-plane are defined by octahedral edges. In producing the same crystalline faces with rhombic dodecahedral CFUs, the a-plane is defined by dodecahedral 4-vertexes and the b-plane and c-plane are defined by dodecahedral faces.