Pentagonal facial joining of hexagonal face icosidodecahedra

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http://web.me.com/whitby/Octahedron/Welcome.html

References

1. Octahedron1stEd.pdf by Robert William Whitby

The octahedral periodicity of the atomic elements and its implications, 500 pages
http://homepage.mac.com/whitby/FileSharing103.html

2. Hexicosidec.pdf by Robert William Whitby

The file shows how identical icosahedral assemblies of twenty carbon atoms each (C_{20}-fullerene) can join hexagonal face to hexagonal face as hexagonal face icosidodecahedra. The resulting forms include the rhombic dodecahedron, a rhombohedron whose faces are the same as the rhombic dodecahedron, the pentagonal dodecahedron, and the icosahedron. Hexagonal face joining also produces the facial planes of the triacontahedron.
http://homepage.mac.com/whitby/BiologicalViruses/FileSharing98.html

3. HexicosidecCube.pdf by Robert William Whitby

The file shows how hexagonal face icosidodecahedra joined hexagonal facially can assemble as five concentric cubes. The faces of the cubes are triacontahedral planes.
http://homepage.mac.com/whitby/BiologicalViruses/FileSharing196.html

4. Hexaconta.pdf by Robert William Whitby

This file shows that hexagonal face icosidodecahedra can join hexagonal face to hexagonal face to form an hexacontahedral assembly.
http://homepage.mac.com/whitby/Quasicrystals/FileSharing197.html

Introduction

This file looks at symmetrical assemblies resulting from the pentagonal face to pentagonal face joining of identical hexagonal face icosidodecahedra. It uses two principal views–(1) normal to an hexagonal face of the base unit and (2) normal to a pentagonal face of the base unit. Two icosidodecahedra joined pentagonal face to pentagonal face are shown in Figure 2. The view is parallel to a pair of opposite pentagonal faces and so taken that the side view of the base unit (See Figure 1.) is parallel to both a pentagonal face and an hexagonal face. In this way, the base unit with its joiner unit can be rotated so that either its hexagonal face or its pentagonal face is normal to the viewing direction. Figure 3 shows the angular rotations required to achieve these orientations.

Struts

The simple assemblies shown in this document can be seen as composed of identical struts. Each strut begins with a base unit and ends with a base unit. The base units are connected by a sin-

gle joiner unit. Each strut has one of six orientations.

When struts are joined in an assembly, one base unit is sufficient for more than one strut. If two struts share a base unit, then one base unit is eliminated; if three struts share a base unit, then two base units are eliminated. If five struts share a base unit then four base units are eliminated.

Triacontahedron

The sixty strut triacontahedron loses forty units for the twenty 3-vertexes and forty-eight for the twelve 5-vertexes. The sixty struts have three units each for 180 units. The sixty strut triacontahedron has 180 units less forty for the 3-vertexes and less forty-eight for the 5-vertexes leaving ninety-two units.

Icosahedron

The icosahedron of twenty 3-hubs loses forty units for the 3-vertexes and one for each of the joins between two hubs for thirty more. The 180 units less the forty for the 3-vertexes and the thirty for the joins between the 3-vertexes leaves 110 units.

Pentagonal dodecahedron

The pentagonal dodecahedron of twelve 5-hubs loses forty-eight for the hubs plus thirty for the joins between the hubs. The 180 units less the forty-eight for the 5-vertexes and the thirty for the joins between the hubs leaves 102 units.

Hexagonal view

For the hexagonal view, there are two joiners–α and γ. The γ-joiner is shown in Figures 4 and 5; the α-joiner is shown in Figures 14 and 15.

γ-join

For each hexagonal face of the base unit there are three pentagonal faces with which it shares an edge. Figure 6 shows the joins between the γ-joiner and each of the three pentagonal faces adjacent to hexagonal face 16 of the base unit. Figure 7 shows the same joins between the γ-joiner and each of the pentagonal faces adjacent to hexagonal face 4. The base unit acts as a threefold hub when a γ-joiner is attached to each of the pentagonal faces adjacent to a hexagonal face. (See Figure 8.) There are twenty hexagonal faces and there are twenty possible hubs. Table 1 lists each of the hub possibilities. The hub number is the number of the hexagonal face that is adjacent to each of the three pentagonal faces listed as γ-joiner sites. Each of the hubs could be a vertex of an acute

Table 1: Hexagonal hubs–acute

Hub	γ-joiner site			Hub	γ-joiner site		
1	A	B	C	11	C	J	K
2	A	C	D	12	D	G	K
3	A	D	E	13	E	G	H
4	A	E	F	14	F	H	I
5	A	B	F	15	B	I	J

Table 1: Hexagonal hubs–acute

Hub	γ-joiner site			Hub	γ-joiner site		
6	C	D	K	16	J	K	L
7	D	E	G	17	G	K	L
8	E	F	H	18	G	H	L
9	B	F	I	19	H	I	L
10	B	C	J	20	I	J	L

Penrose rhombohedron. [See Reference 1, page 124—QUASICRYSTAL:Acute Penrose rhombohedron.]

Pyramid

Figures 14 and 15 show two views of a pyramidal assembly consisting of four γ-hubs.

α-join

Three additional pentagonal faces are related to a view normal to an hexagonal face of the base unit. Each is separated from the central hexagonal face by another hexagonal face. [See Figure 17.] When an α-joiner is attached to each of the three faces as in Figure 19, the base unit acts as a threefold hub. The hub could be a vertex of an obtuse Penrose rhombohedron.

Table 2: Hexagonal hubs–obtuse

Hub	α-joiner site			Hub	α-joiner site		
1	D	F	J	11	B	D	L
2	B	E	K	12	C	E	L
3	C	F	G	13	D	F	L
4	B	D	H	14	B	E	L
5	C	E	I	15	C	F	L
6	A	G	I	16	C	G	I
7	A	H	K	17	D	H	J
8	A	G	I	18	E	I	K
9	A	H	J	19	F	G	J
10	A	I	K	20	B	H	K

Triacontahedron

A face of the triacontahedron is geometrically identical to a face of a Penrose rhombohedron. Figure 23 shows a triacontahedron viewed parallel to a 3-vertexial diameter. Figure 24 shows how eight hexagonal face icosidodecahedra joined pentagon to pentagon form the edges of a face of a triacontahedron. The edgial diameter 10,1-13,18 of each unit is parallel to the viewing direction. The units at the four vertexes are identically oriented. The units which connect the vertexes have two orientations which are rotated about the viewing direction relative to the vertexial units. The connecting unit one edge has the same orientation as the connecting unit in a parallel edge. Each join of the facial assembly is parallel to the viewing plane.

Ten faces of the triacontahedron are parallel to a given 5-vertexial diameter. These faces can be extended parallel to the 5-vertexial diameter without affecting their spatial orientation or that of any of the other twenty faces. Figure 25 shows how this extension can be effected in a triacontahedral assembly of icosidodecahedra.

Figure 26 shows an hexagonal ring of α-joined icosidodecahedra and two α-joined hubs. Joining a hub to the ring produces three faces of a triacontahedron at a threefold vertex. [See Figure 25.] Each of the three-face assemblies is extended to produce a twelve face assembly [Figures 27 and 28] which are then joined as a complete triacontahedral assembly [Figure 30]. The joining merges the six outermost base units. There is a base unit at each of the twelve 5-vertexes; there is a base unit at each of the twenty 3-vertexes; and there are sixty units connecting the thirty-two vertexes. The total number of icosidodecahedra in the complete assembly is ninety-two.

Each rhombohedral face of a triacontahedron that is composed of identical hexagonal face icosidodecahedra is bounded by two of the six edges which result from the pentagonal face to pentagonal face joining. Table 2 shows each of the ten faces which include a given edge.

Table 3: Triacontahedral edges and faces

Edge	Faces									
AL	7	13	8	14	9	15	10	11	6	12
BG	19	20	16	11	6	2	3	4	8	14
CH	17	12	7	3	4	5	9	15	20	16
DI	8	4	5	1	10	11	16	17	18	13
EJ	17	18	19	14	9	5	1	2	6	12
FK	2	3	7	13	18	19	20	15	10	1

Triacontahedron–elongated

Figures 32 through 34 show the assembly of an elongated triacontahedron. The views herein are the same as those used for the triacontahedron. The number of units is increased by the thirty units used to extend the ten faces for a total of 122 units.

Icosahedron

Two γ-hubs can be linked by a base unit. All twenty γ-hubs can be linked by a base unit to three other γ-hubs to form an icosahedral assembly. Figures 35 and 36 show each of the γ-hubs. Figure 38 shows the lower portion of the assembly; Figure 39 shows the mid-layer; Figure 40 shows the

lower portion joined with the mid-layer; Figure 41 shows the upper portion of the assembly; and Figure 42 shows the complete assembly.

There are twenty hubs. [See Table 1.] Each hub contains one base unit and three joiners which account for eighty units. There are thirty base units connecting the hubs. Altogether, there are 110 units in the assembly.

Pentagonal view

β-join

Figure 44 shows a β-joiner with a base unit. Figure 45 shows how face D of the β-joiner can join with the G, H, I, J, and K faces of the base unit to form the β-hub L of Figure 46.

Base unit joins with β-joiner face I are shown in Figure 47. Figure 48 shows β-hub A.

Pentagonal dodecahedron

There are twelve pentagonal β-hubs. Table 4 shows the joiner sites for each of the hubs. The hubs can be joined in an assembly so that each hub lies at the center of a face of a pentagonal dodecahedron [See Figure 49 for an oblique view of a pentagonal dodecahedron.] Figures 51 and 52 show each of the twelve β-hubs. Figures 52 through 55 show how the hubs are joined together by additional base units to form the complete pentagonal dodecahedral assembly shown in Figure 56.

Table 4: Pentagonal hubs

Hub	β-joiner site				
A	B	C	D	E	F
B	A	C	F	I	J
C	A	B	D	J	K
D	A	C	E	G	K
E	A	D	F	G	H
F	A	B	E	H	I
G	D	E	H	K	L
H	E	F	G	I	L
I	B	F	H	J	L
J	B	C	I	K	L
K	C	D	G	J	L
L	G	H	I	J	K

Triacontahedron

The triacontahedral assembly shown previously in the hexagonal view is shown here in a pentagonal view. Figures 61 through 66 show the progress of the assembly. The complete assembly is shown in Figure 67.

Elongated triacontahedron

In the pentagonal view used here, the viewing direction is parallel to the pentagonal axis of the elongated triacontahedron. The upper and lower assemblies of the elongated triacontahedron are the same as their counterparts in the triacontahedron. The middle assembly of the elongated triacontahedron is composed of three rings [See Figure 68.] Rings A and C are identical to each other and to the mid-layer of the triacontahedron. An enlarged view of ring B is shown in Figure 69 and the completed middle assembly ABC is shown in Figure 70. The view of the complete elongated triacontahedron is the same as that of the complete triacontahedron of Figure 67.

Pentagonal ring

Figures 71 and 72 show opposite sides of the same pentagonal ring. The ring consists of ten base units and ten joiners. There are twelve such rings in the icosahedral assembly of Figure 42, one for each pentagonal face. Figures 40 and 41 show two views of a three ring junction.

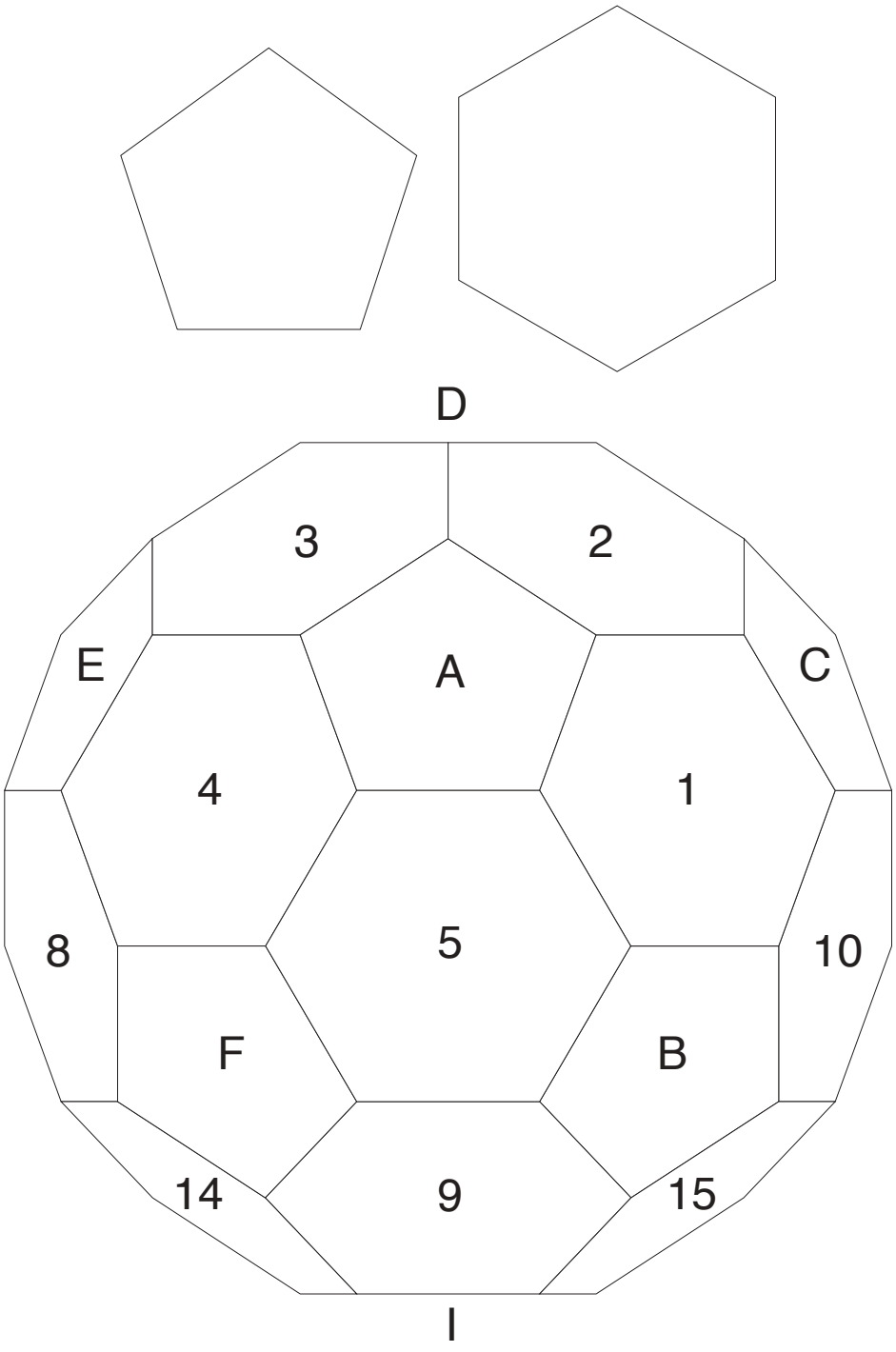


Fig. 1 Hexagonal face icosidodecahedron–view normal to pentagonal facial diameter DI

The figure shows an hexagonal face icosidodecahedron. The view is perpendicular to the diameter between the pentagonal faces D and I. Both a pentagonal face and an hexagonal face are shown in true projections at the top of the figure.

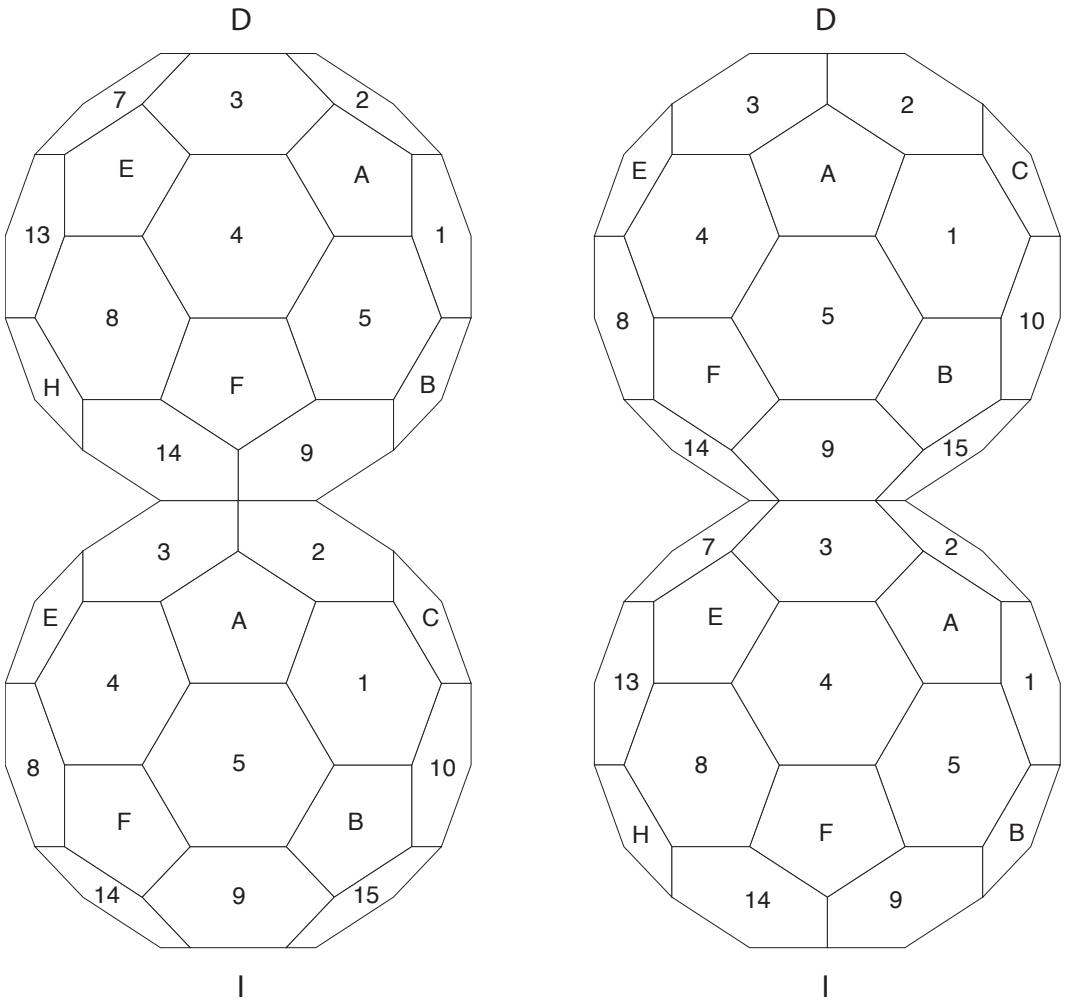
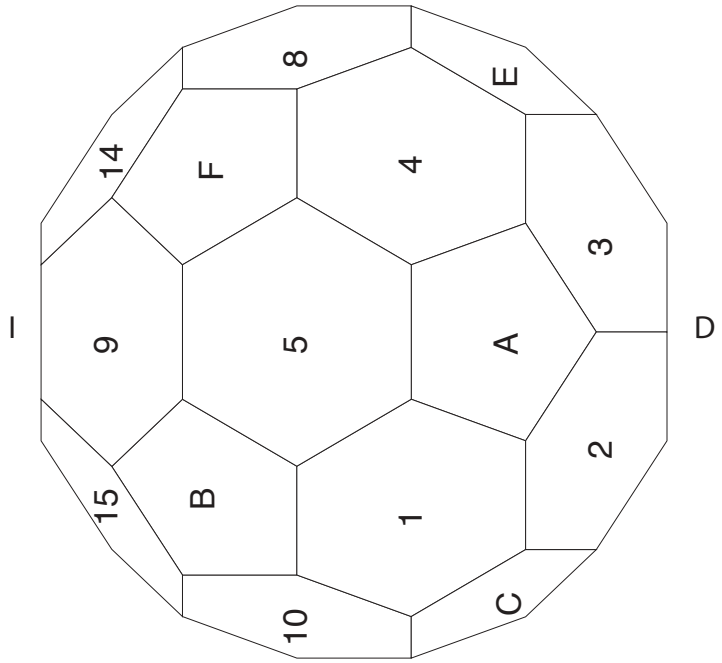


Fig. 2 Hexagonal face icosidodecahedra–pentagonal facial joining

The figure shows two pentagonal facial joins between identical hexagonal face icosidodecahedra. The adjoined units are rotated relative to one another by one-tenth turn about the join axis which is perpendicular to the adjoined faces.

On the left, the I-face of the upper unit is congruent with the D-face of the lower unit.

On the right, the positions of the units are reversed so that the D-face of the lower unit is congruent with the I-face of the upper unit.



α = atn{1/[3+sqr(5)]} ~10.8123 deg β= atn(1/2) ~ 26.5651 deg

γ= atn(sqr{[7+3*sqr(5)]/8})

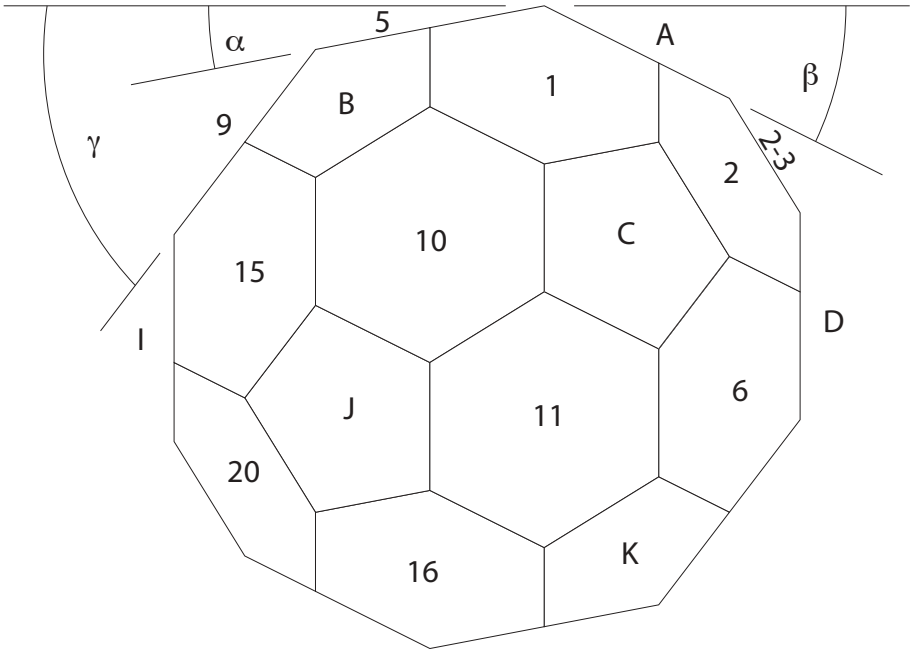


Fig. 3 Rotations of base unit to *hexagonal face up* and *pentagonal face up* orientations

Hexagonal view

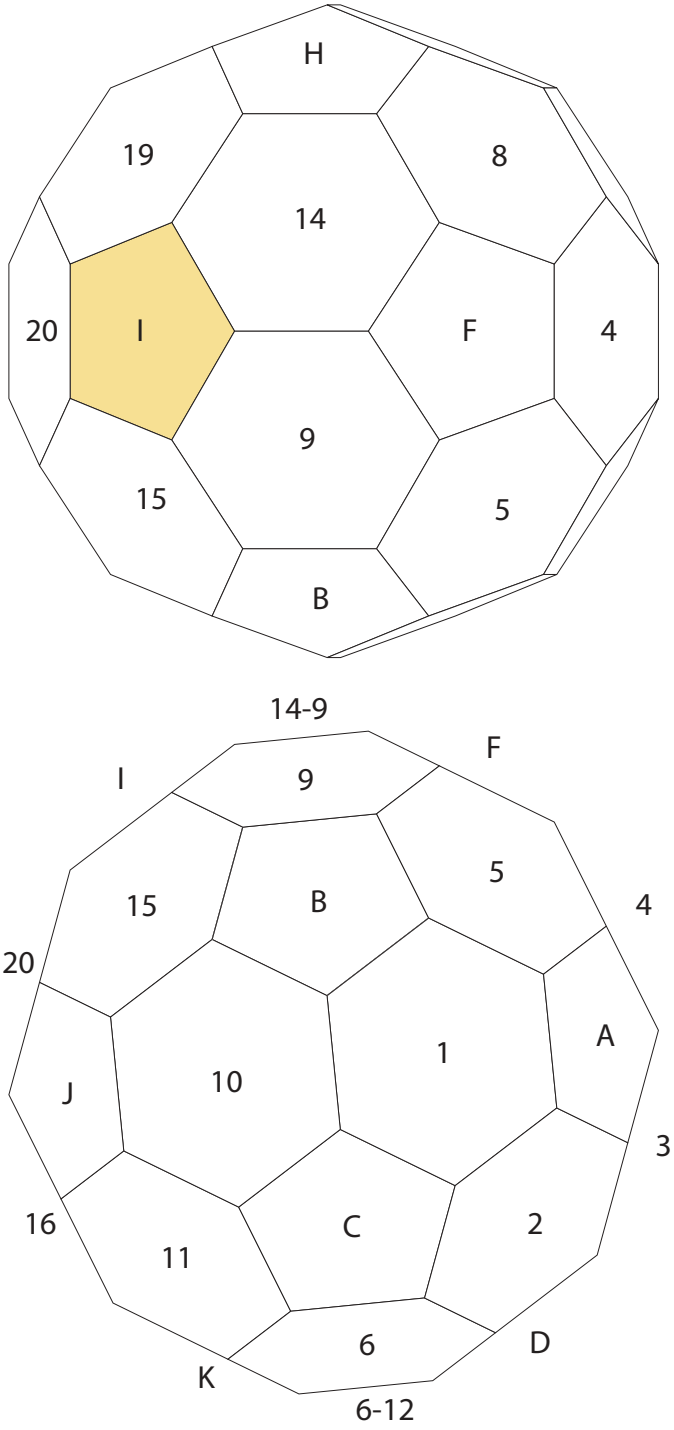


Fig. 4 Joiner unit for base unit rotation γ

The figure shows two views of an hexagonal face icosidodecahedron that is oriented so that it can be joined to an identical icosidodecahedron that is oriented so that an hexagonal face is parallel to the projection plane. When rotated about a normal to the projection plane, the pentagonal face that is colored yellow can make a join with three of the faces of the other unit.

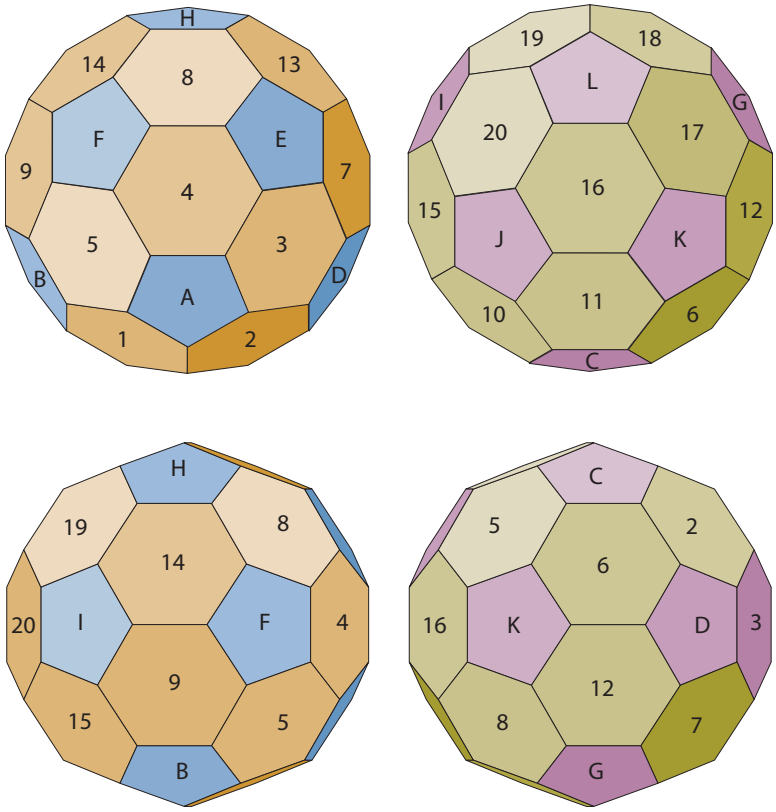


Fig. 5 Hexagonal view of base unit with γ-joiner unit

On the left, the figure shows two hexagonal face icosidodecahedra oriented so that they can be joined pentagonal face to pentagonal face.

On the right, the faces on the underside of each of the icosidodecahedra are shown as they appear when looking from the inside toward the outside.

The lower unit can be rotated about the viewing direction so that its I face can join to the J, K, or L face of the upper unit and its D face can be joined to the A, E, or F face of the upper unit.

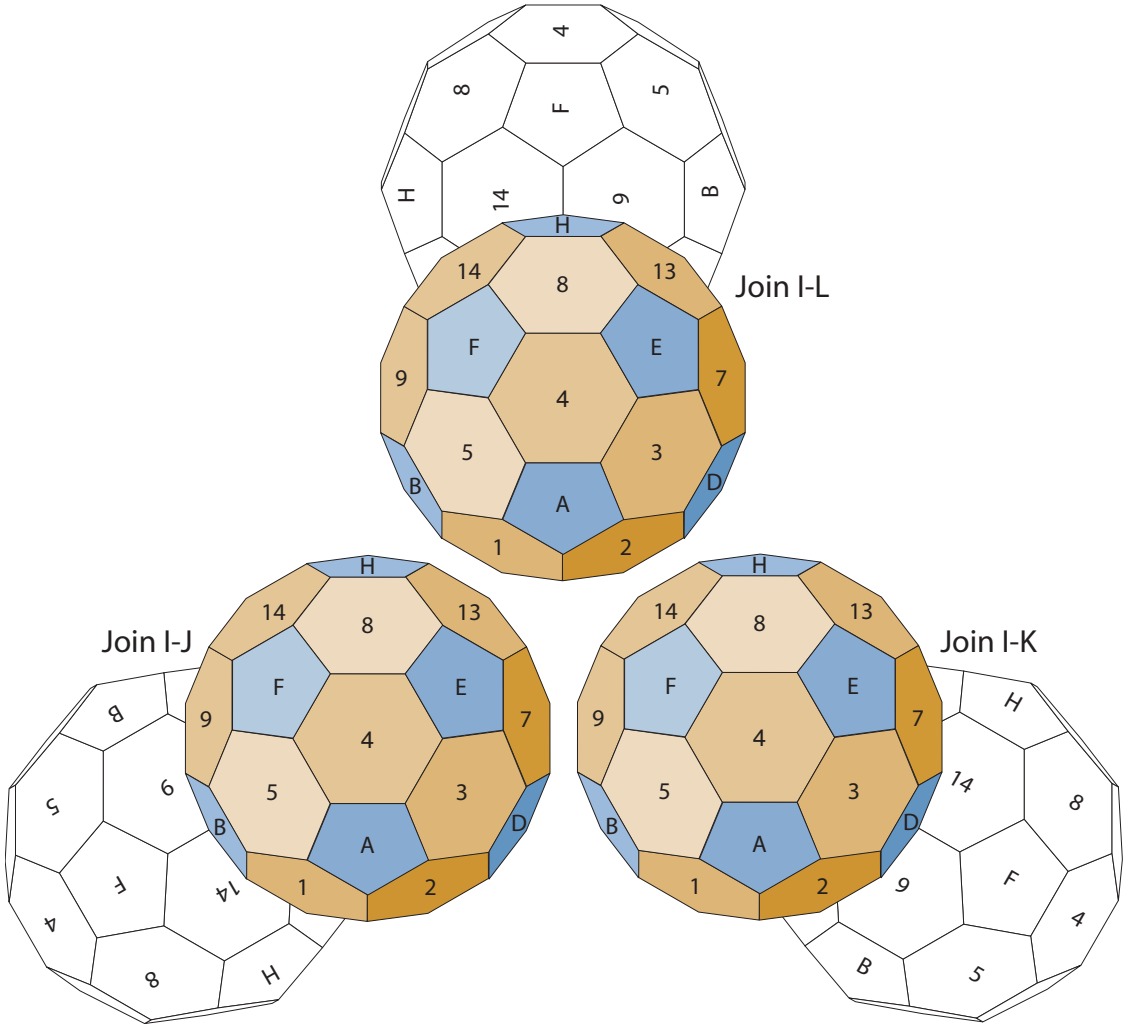


Fig. 6 Pentagonal joins between γ-joiner face I and base faces J, K, and L
The figure shows the joins between base unit faces J, K, and L and joiner unit face I viewed normally to hexagonal face 4.

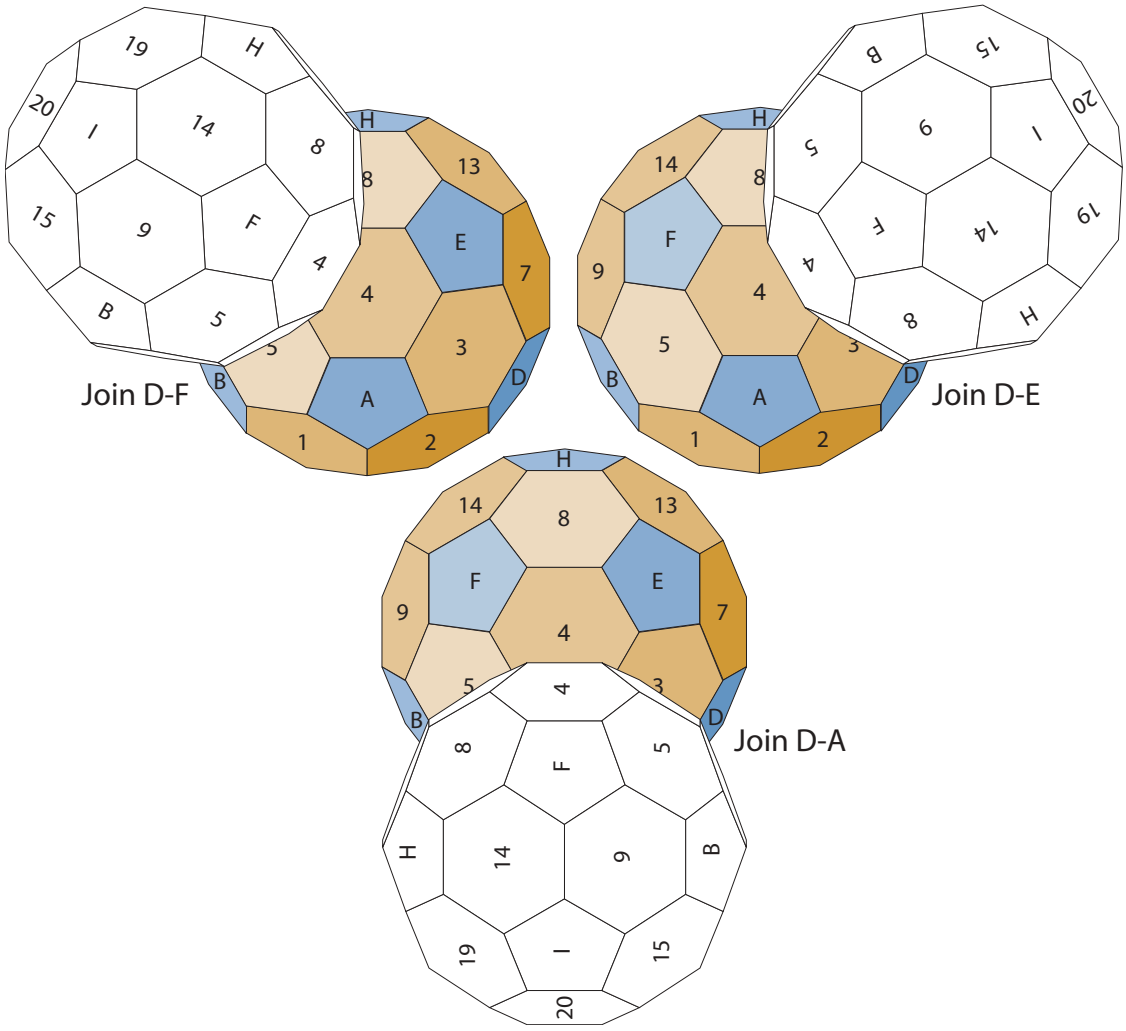


Fig. 7 Pentagonal joins between γ-joiner face D and base faces A, E, and F

The figure shows the joins between base unit faces A, E, and F and joiner unit face D viewed normally to hexagonal face 4.

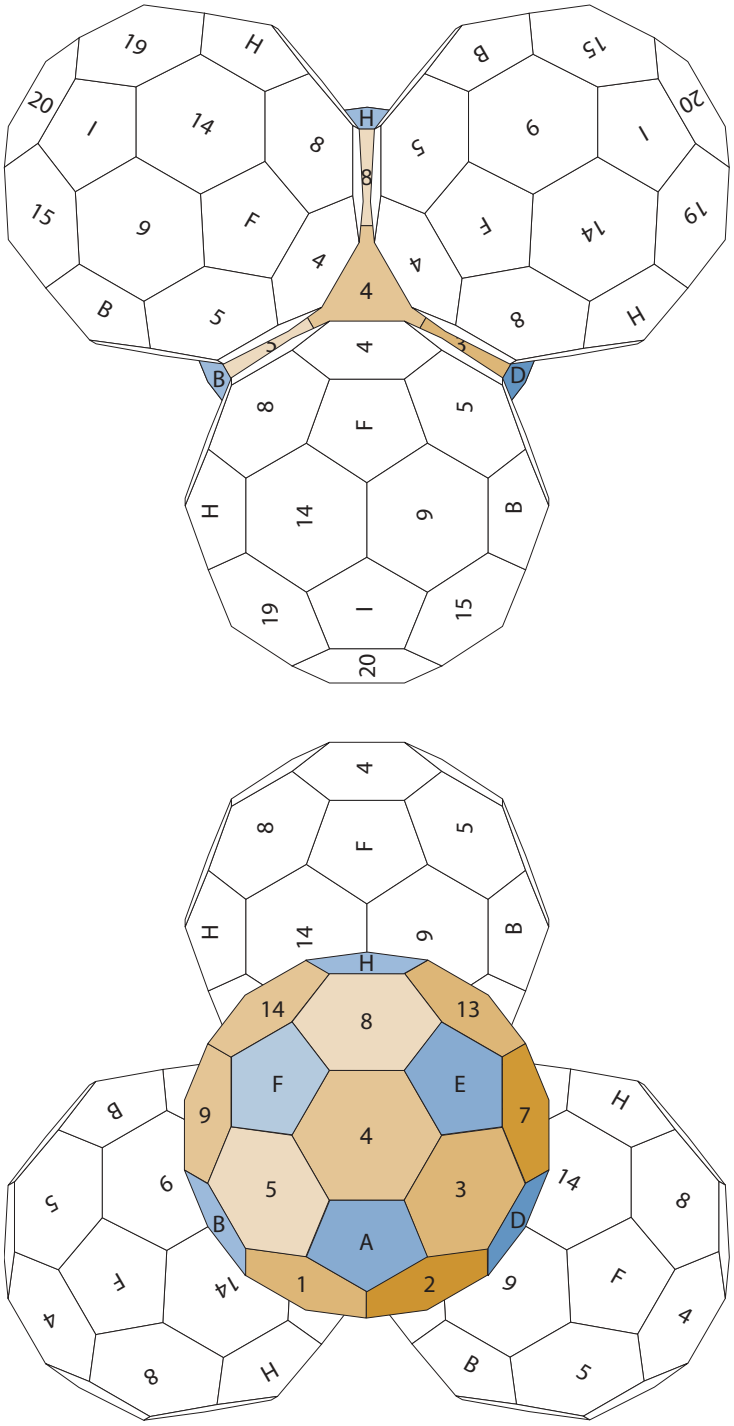


Fig. 8 Two γ-hubs

Base unit with γ-joiner units on faces J, K, and L and base unit with γ-joiner units on faces A, E, and F

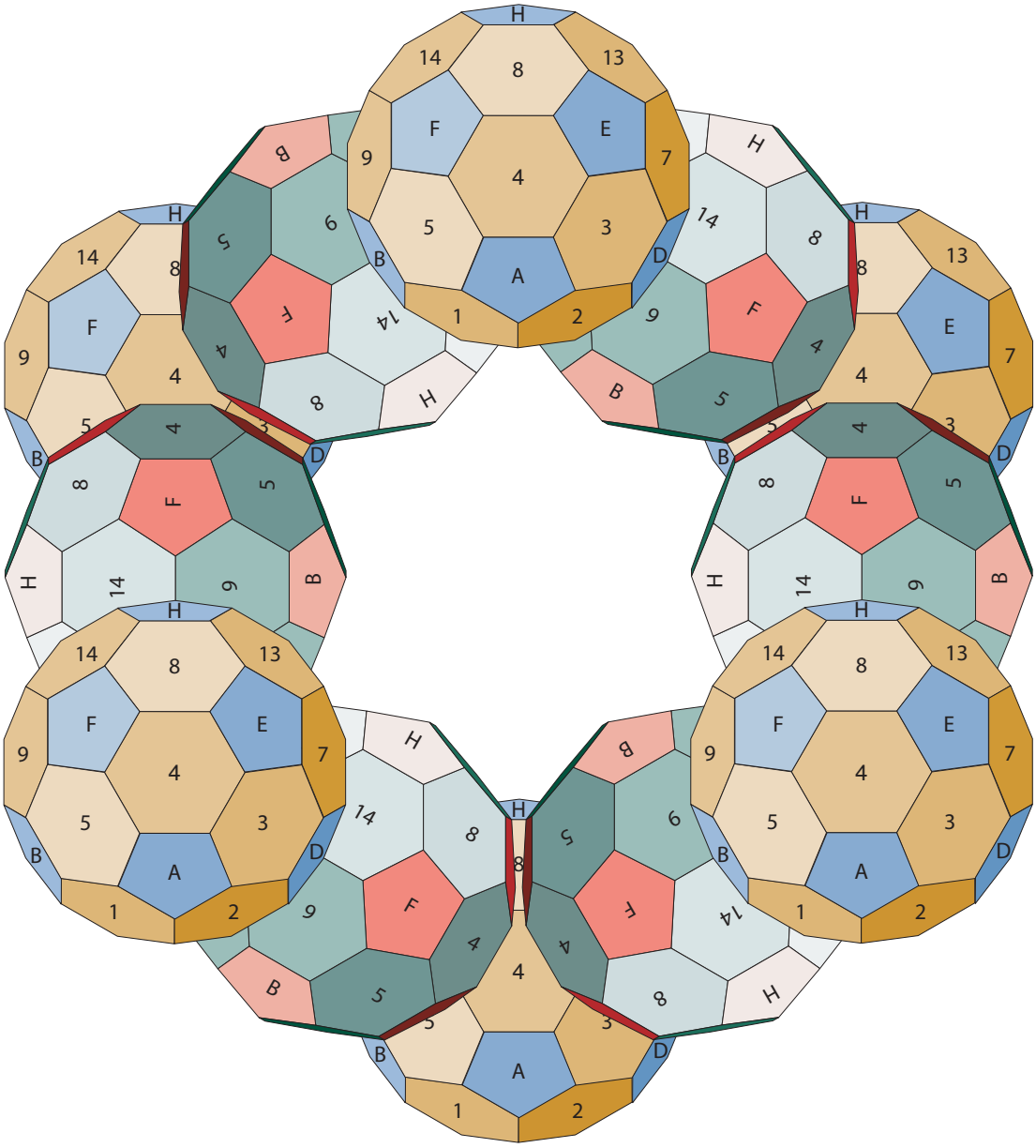


Fig. 9 Hexagonal ring of γ-joiners and base units

The ring is composed of six linear segments each consisting of a pair of base units joined by a γ-base unit. Each of the six pairs of adjoining linear segments defines one of the six faces of an acute Penrose rhombohedron. Each of the base units is at one of the middle vertexes of the rhombohedron.

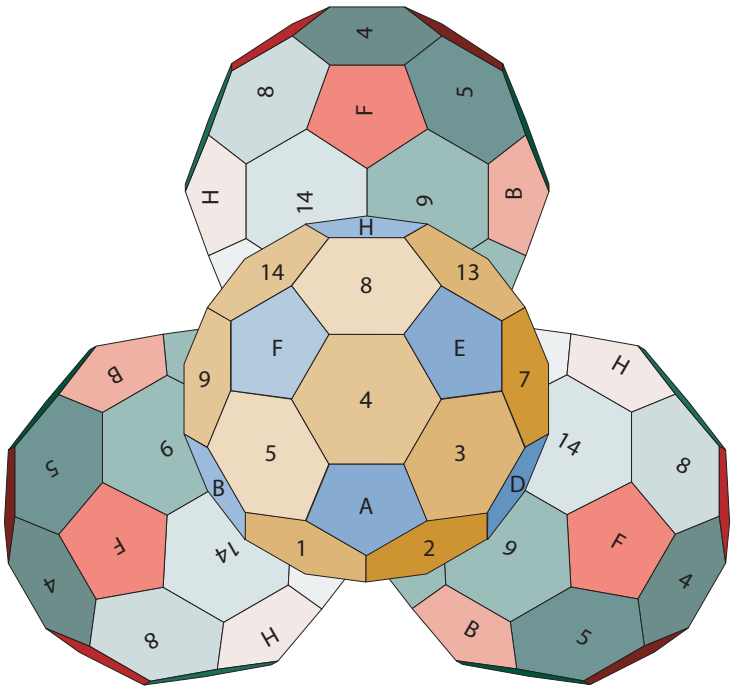


Fig. 10 Base unit with three γ-joiners

Each of the twenty hexagonal faces of the icosidodecahedron shares an edge with three pentagonal faces. With an γ-joiner on each of the faces, the threefold assembly shown in the figure results. An icosahedral assembly can be formed which uses an identical assembly for each of the twenty hexagonal faces. Each joiner of each assembly is joined to a base unit. Each of these base units is joined to another hub.

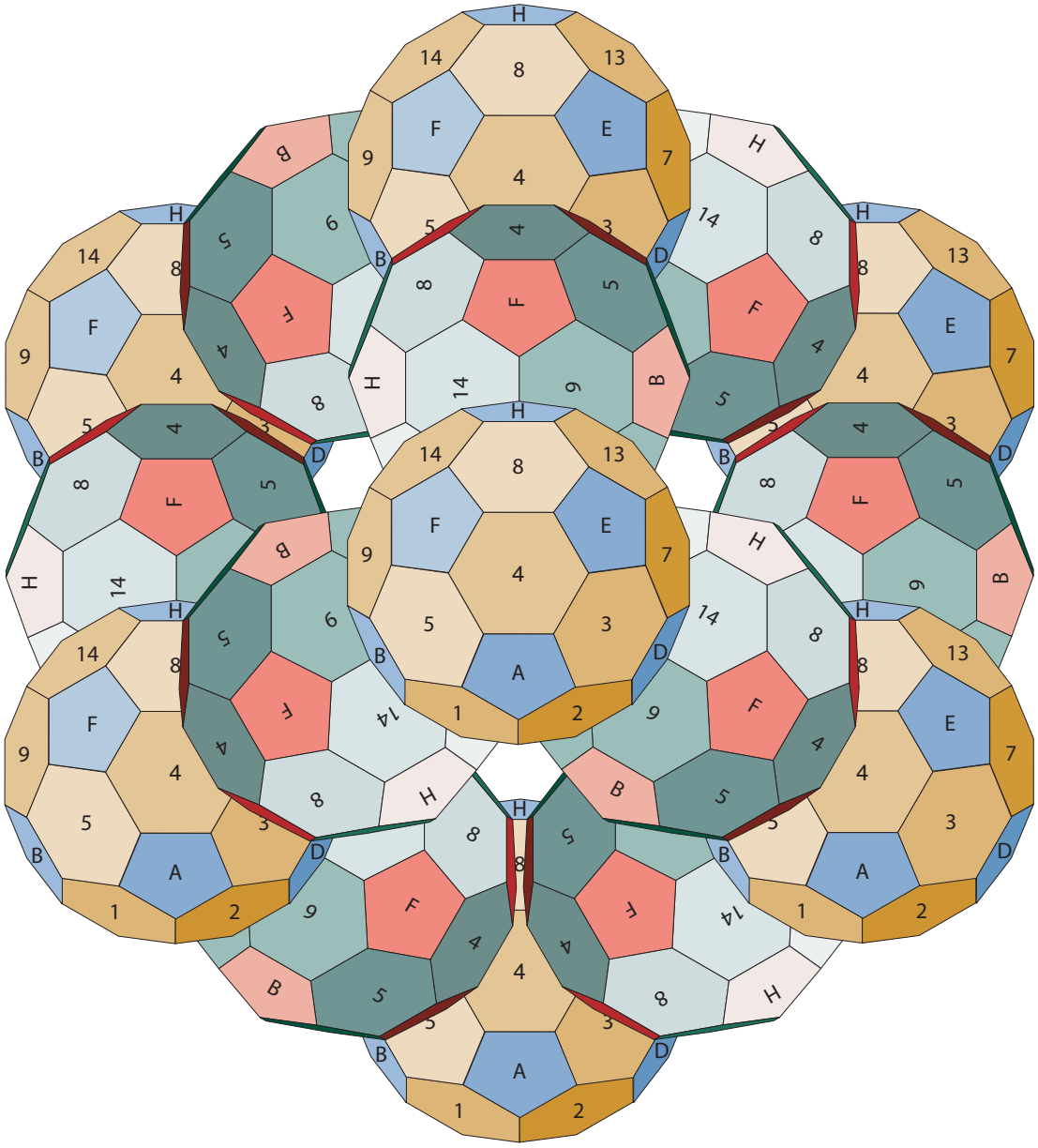


Fig. 11 γ-joined hexagonal ring with threefold cap

The hexagonal ring of Figure 9 is capped by the assembly of Figure 10. The cap completes the edges and vertexes of three acute Penrose rhombohedral faces.

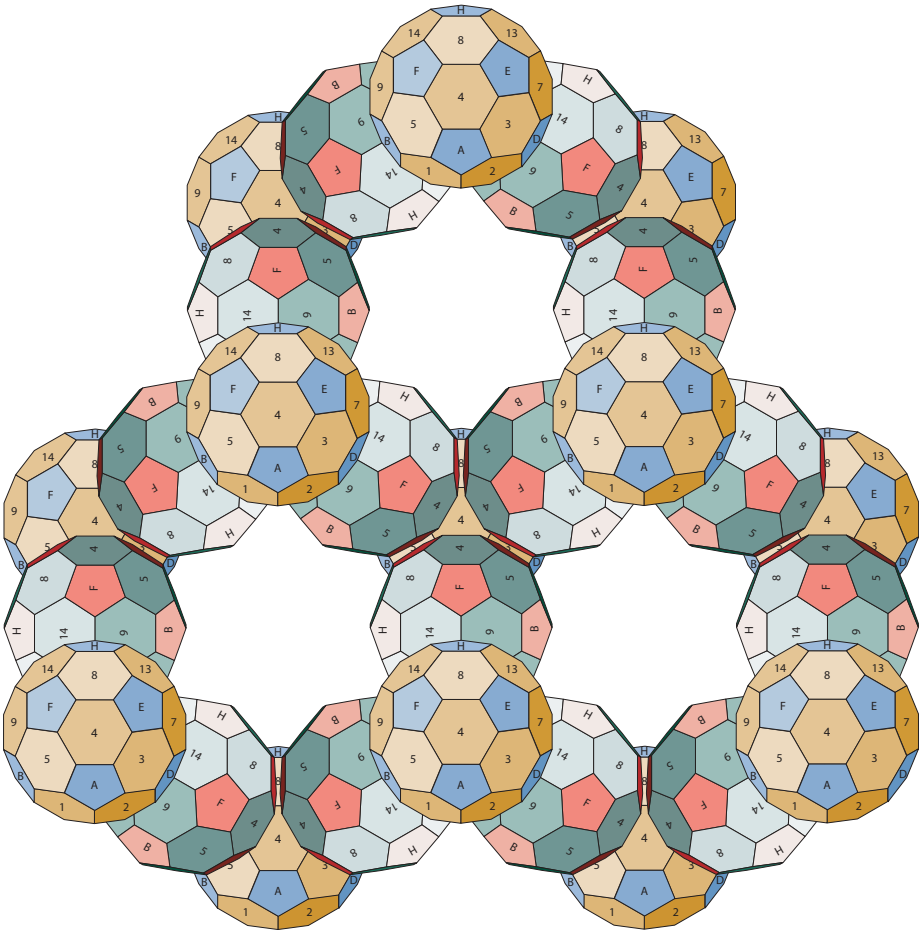


Fig. 12 Three merged hexagonal rings of γ-joiners and base units

The figure shows an assembly of three merged rings. Each ring shares a linear segment with each of the other rings. There are three sets of five parallel linear segments

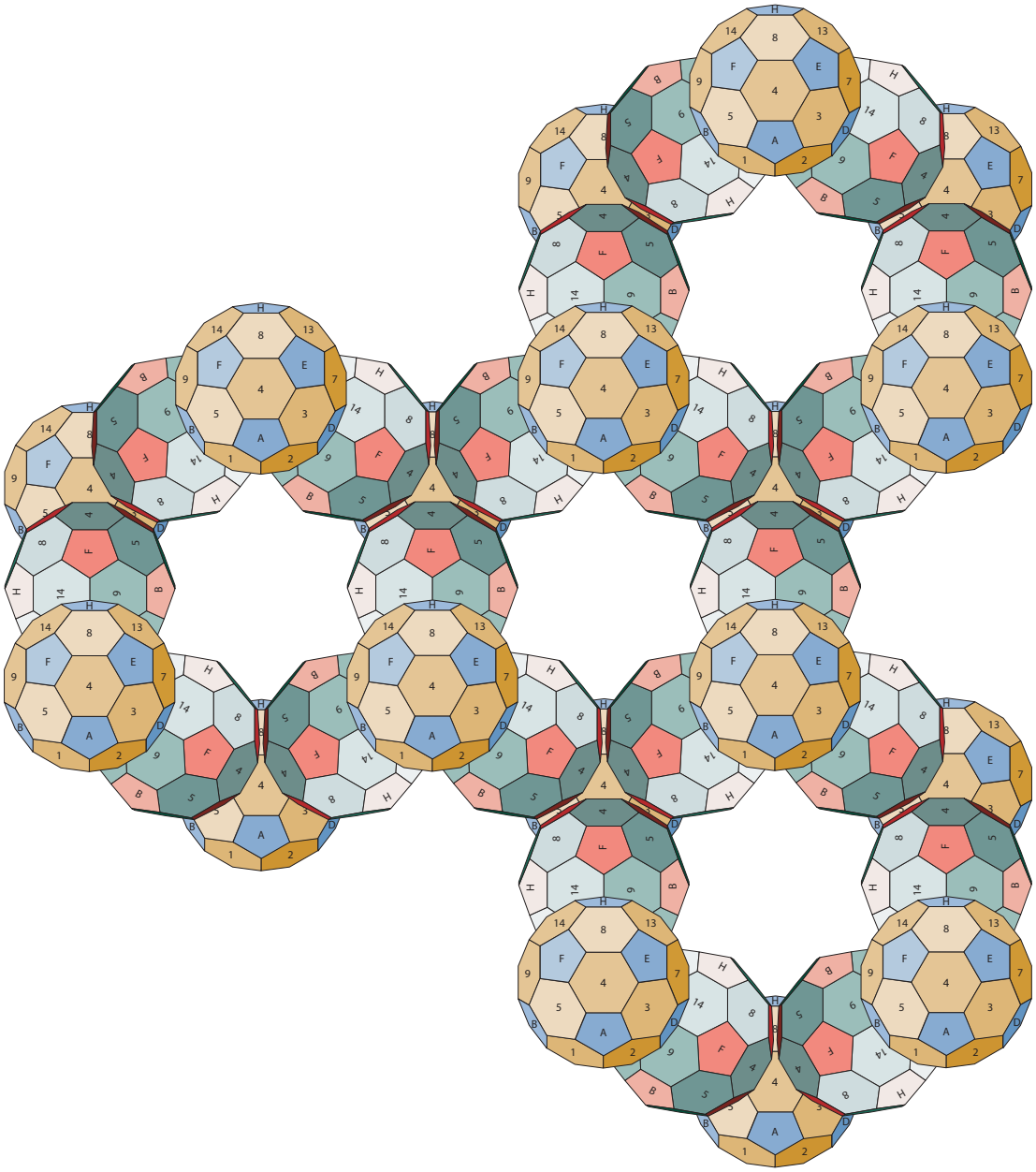


Fig. 13 Four merged hexagonal rings of γ-joiners and base units

The figures shows an assembly consisting of four merged rings. The central ring shares a linear segment with each of the three outer rings. There are three sets of seven parallel linear segments.

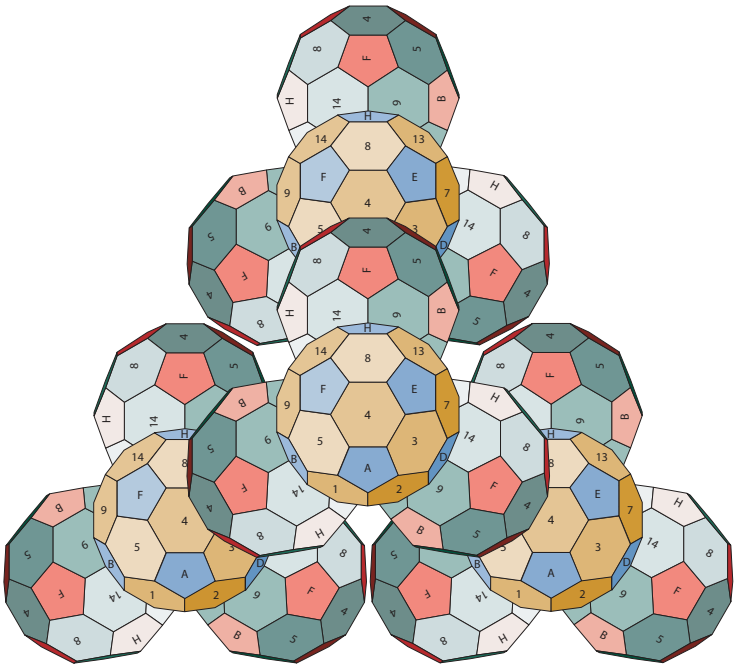


Fig. 14 Pyramidal assembly of four γ-hubs–vertexial view

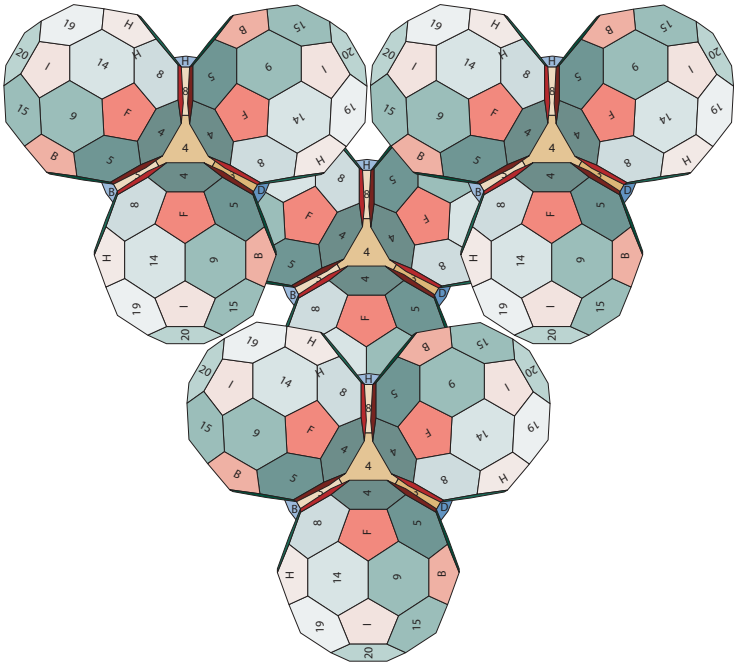


Fig. 15 Pyramidal assembly of four γ-hubs–base view

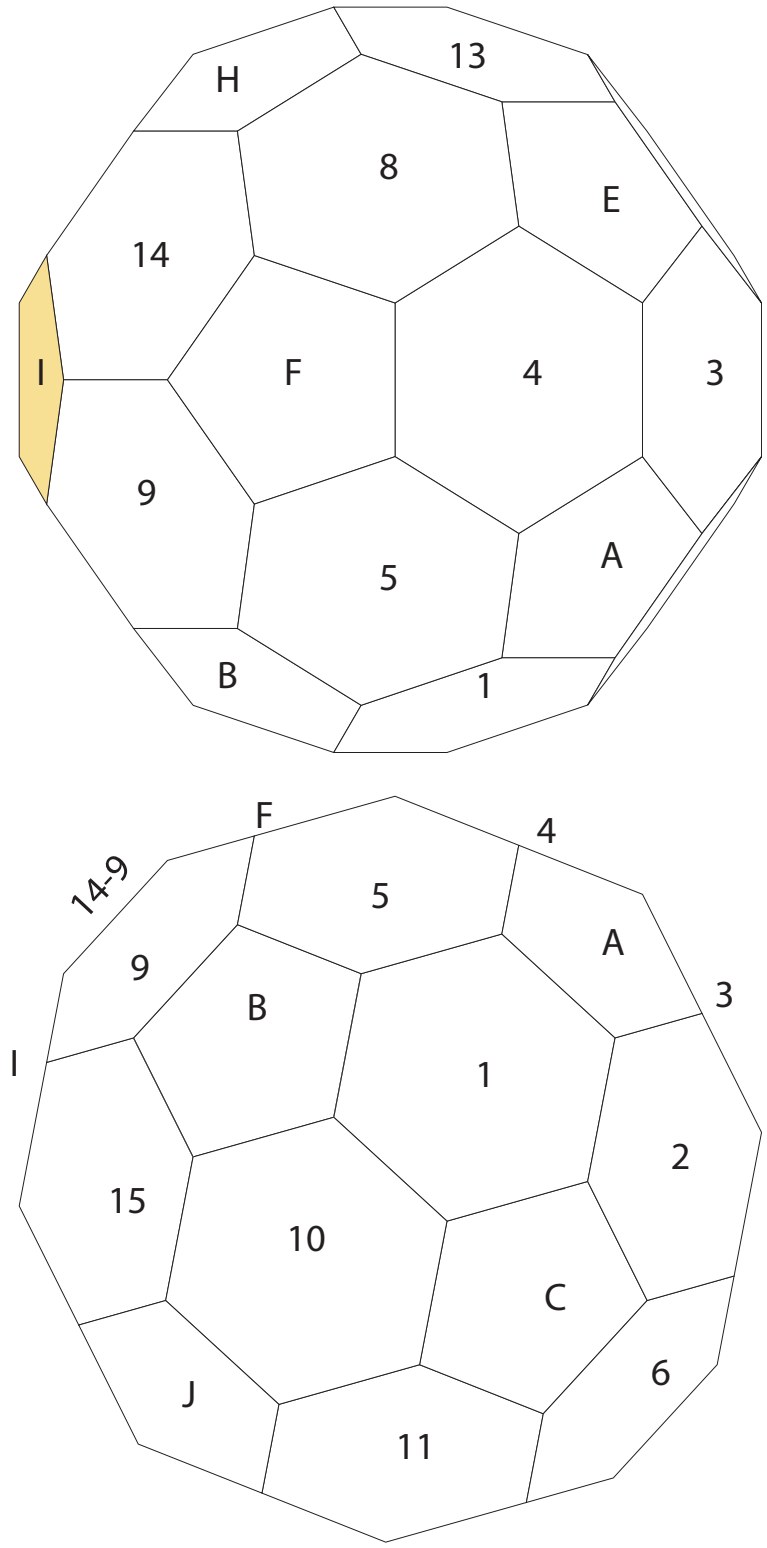


Fig. 16 α-joiner unit

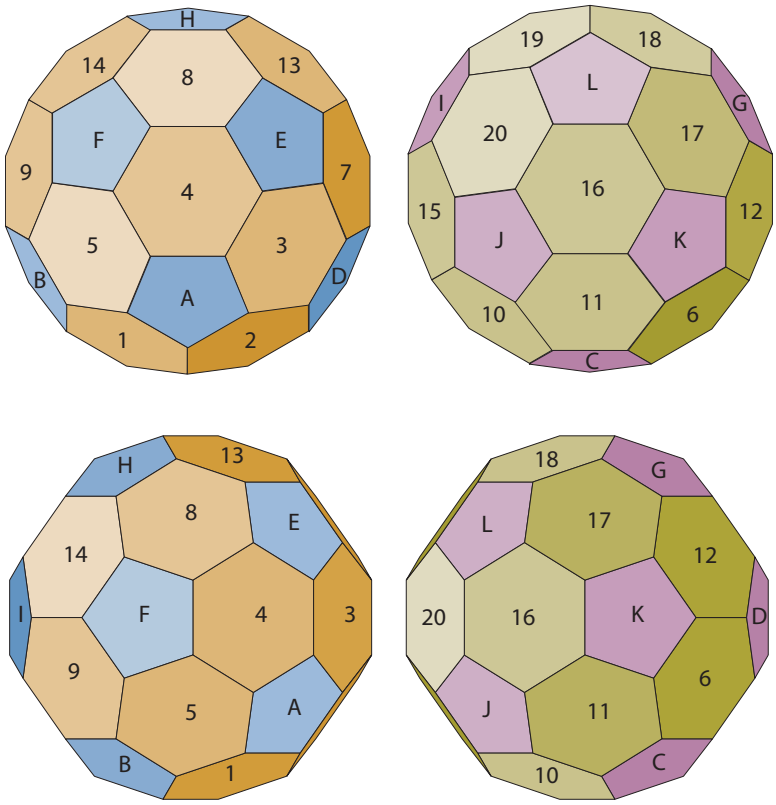


Fig. 17 Hexagonal view of base unit with α-joiner

The figure shows a hexagonal face view with the joiner for the base rotation α. When rotated about an axis normal to the projection plane, the I-face of the joiner can join with the C, G, and I faces of the hub unit, and the D-face of the joiner can join with the B, D, and H faces of the hub unit.

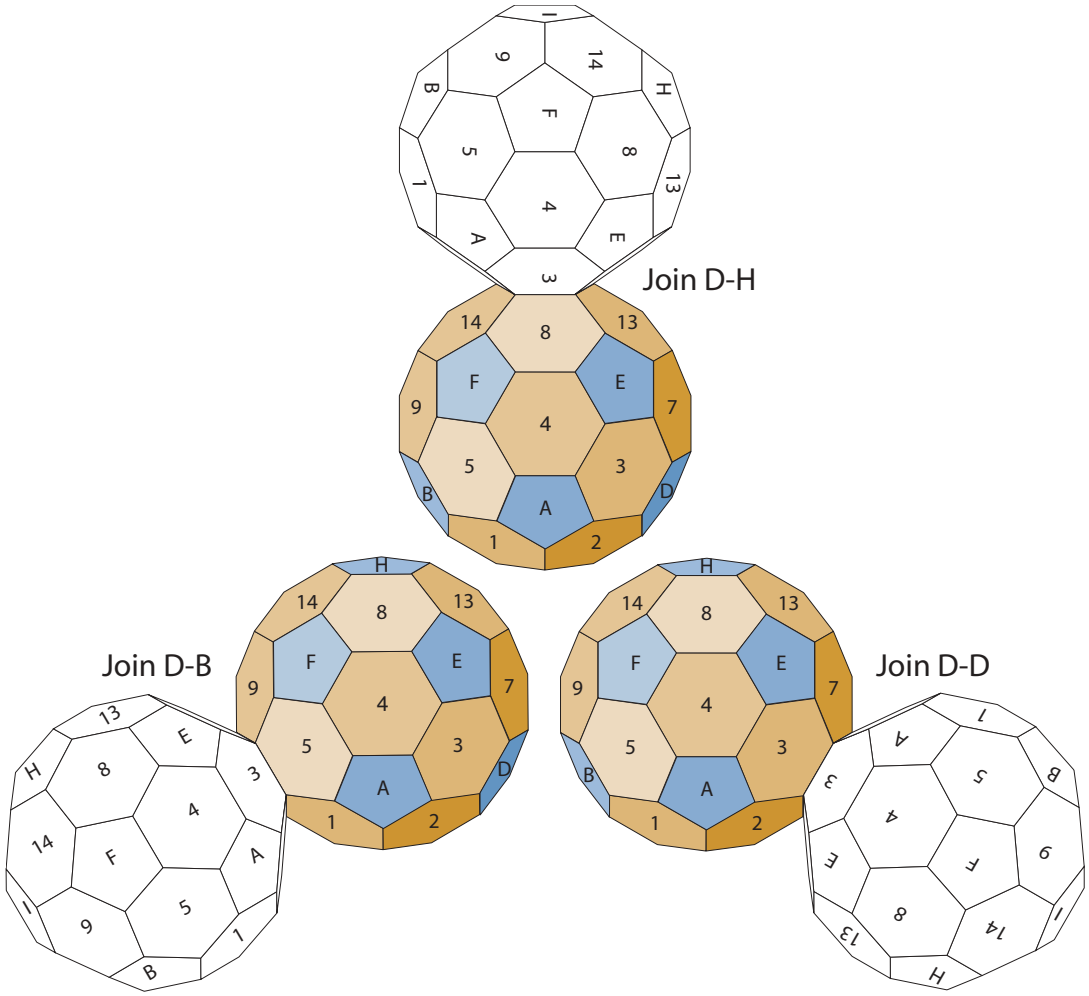


Fig. 18 Three base units with an single α-joiner on face B, D, or H

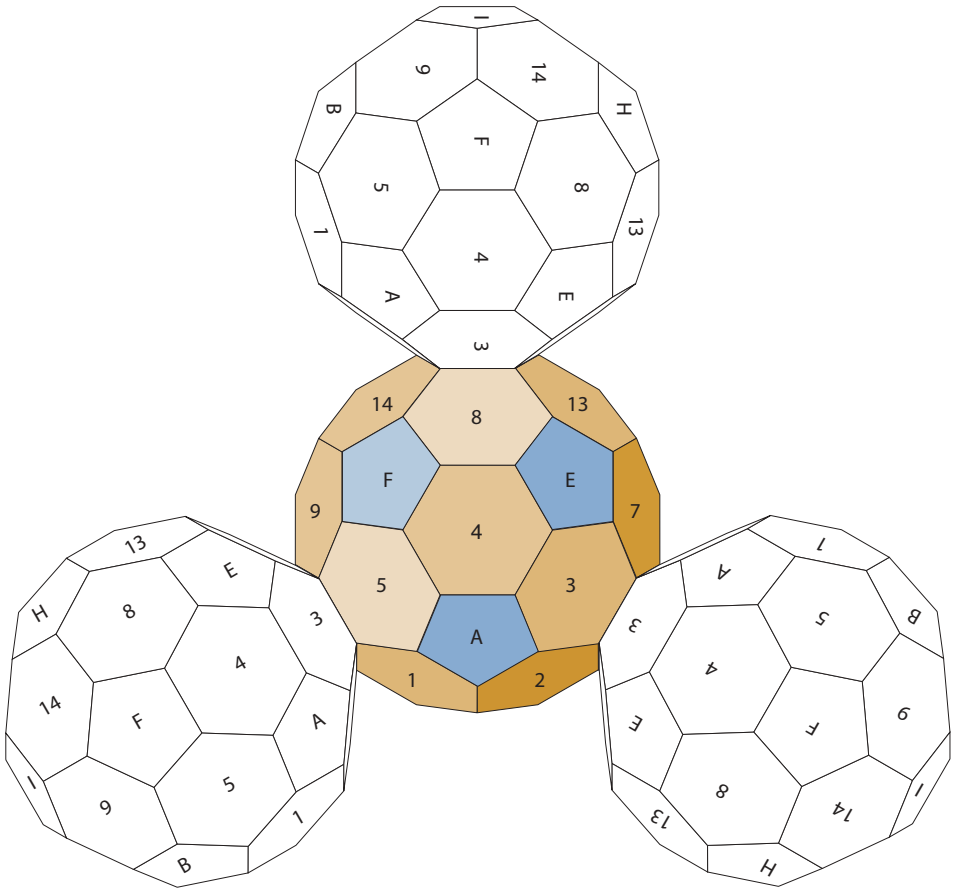


Fig. 19 One base unit with an α-joiner on faces B, D, and H

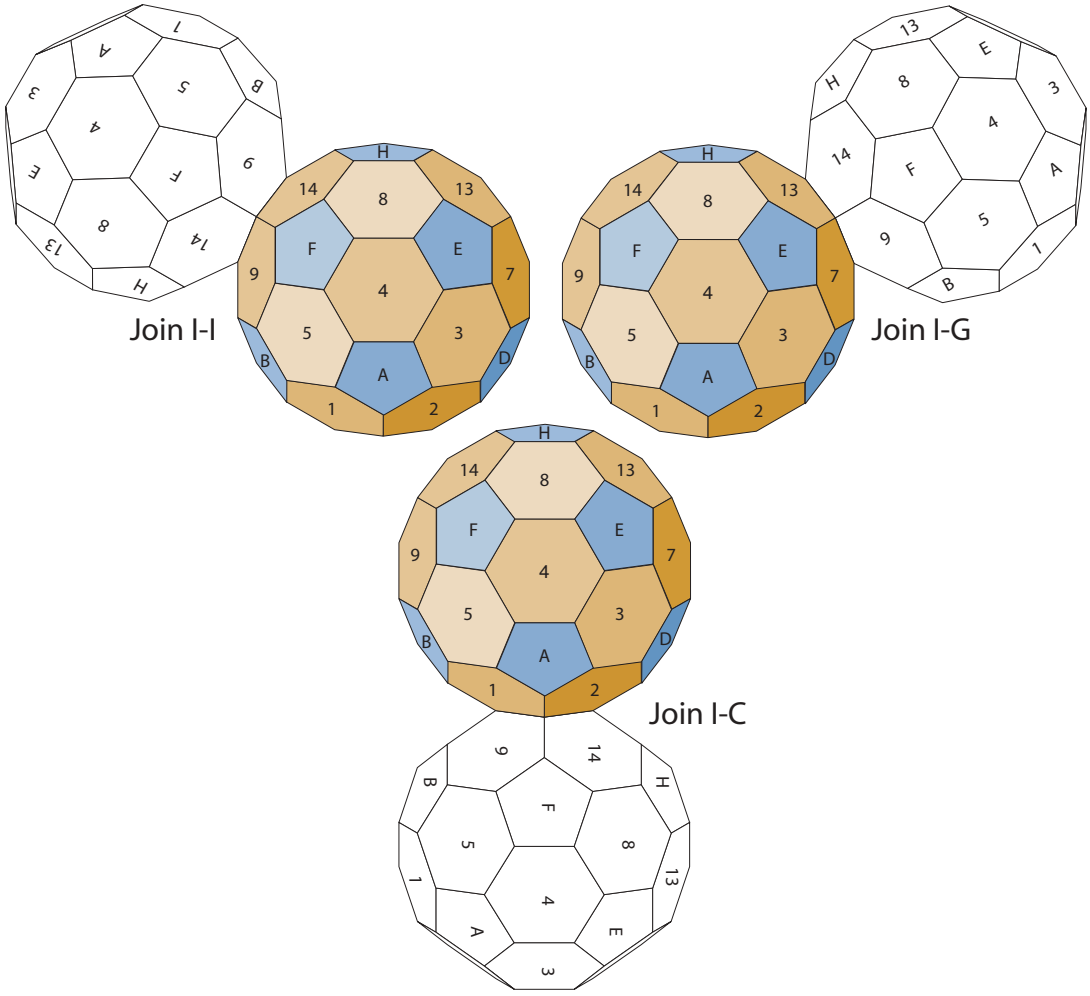


Fig. 20 Three base units with a single α-joiner on face C, G, or I

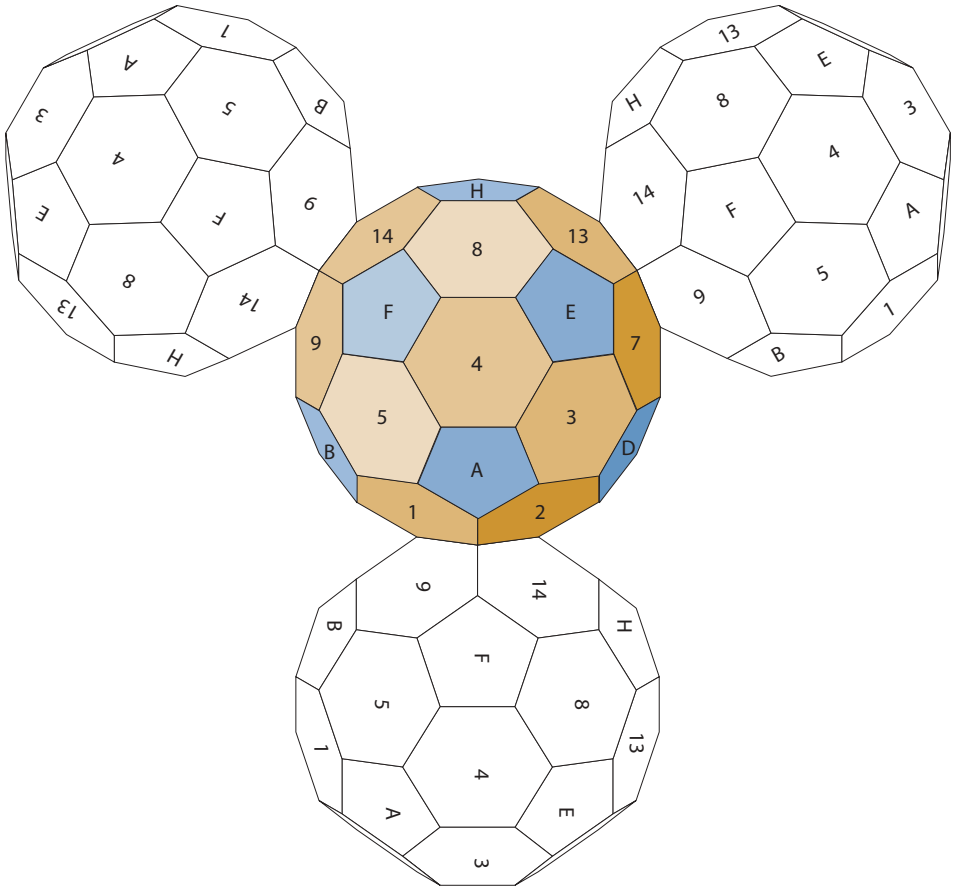


Fig. 21 One base unit with an α-joiner on faces C, G, and I

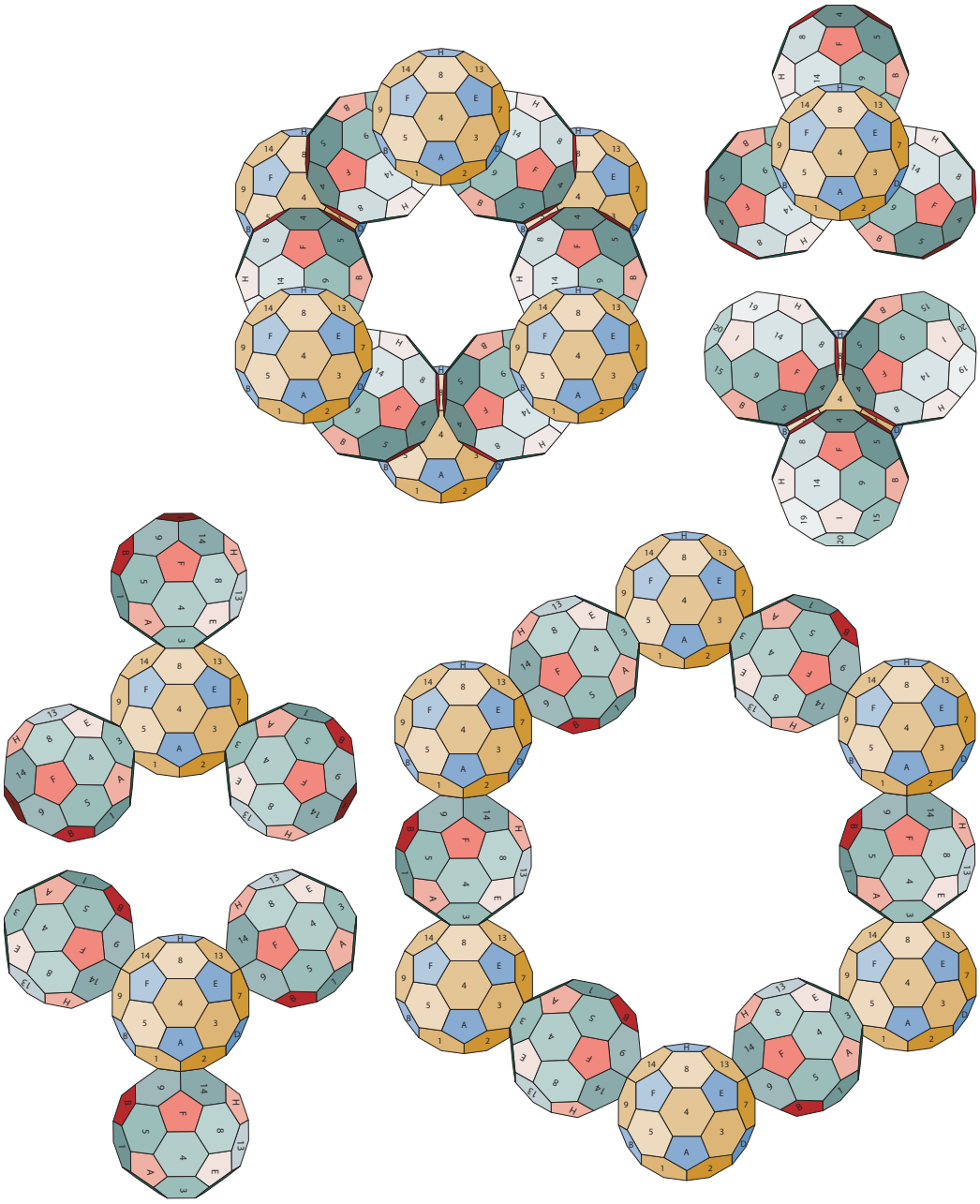


Fig. 22 Hexagonal rings with threefold hubs

At top, an hexagonal ring of base units and γ-joiners is on the left and two threefold γ-joiner hubs are on the right. These are the components of an acute Penrose rhombohedron.

At bottom, an hexagonal ring of base units and α-joiners is on the left and two threefold α-joiner hubs are on the right. These are the components of an obtuse Penrose rhombohedron.

Triacontahedron

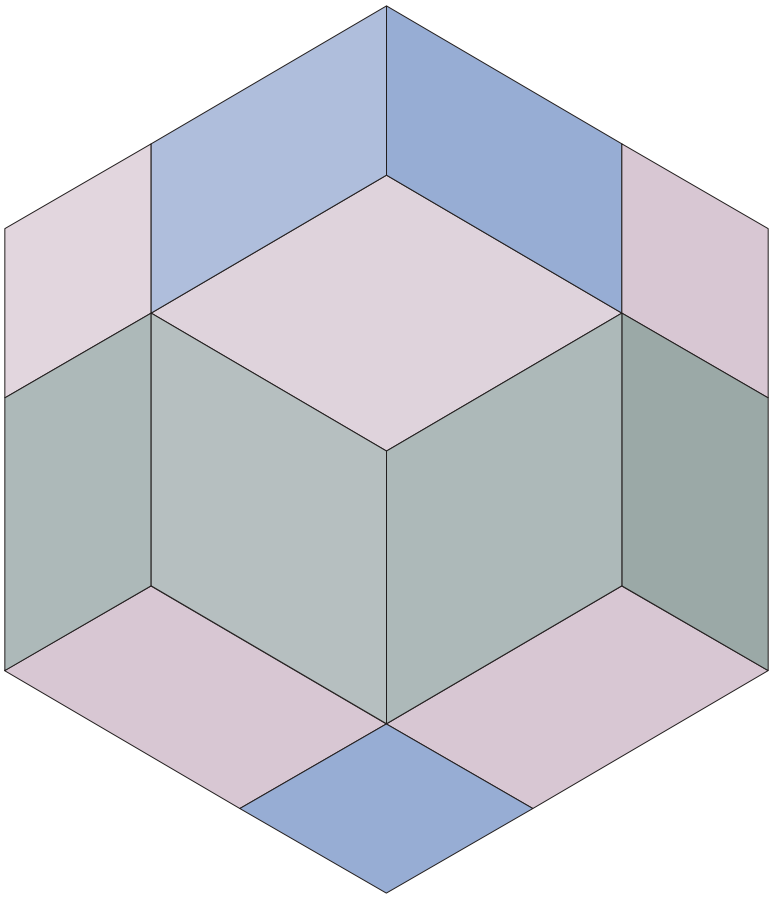


Fig. 23 Triacontahedron viewed parallel to a 3-vertexial diameter

The triacontahedron shown in the figure is viewed parallel to one of its ten 3-vertexial diameters. The diameter is equivalent to a facial diameter of a regular icosahedron. The blue faces share a 5-vertex on a fivefold axis; the gray faces are parallel to the fivefold axis; the violet faces are intermediate between the violet and the gray faces. There are two groups of five blue faces (three showing), two groups of violet faces (five showing); and ten gray faces (four showing). The hexagonal boundary of the figure is composed of six faces that are parallel to the viewing direction–one gray face on the left, one gray face on the right, two blue faces at the top, and two blue faces at the bottom.

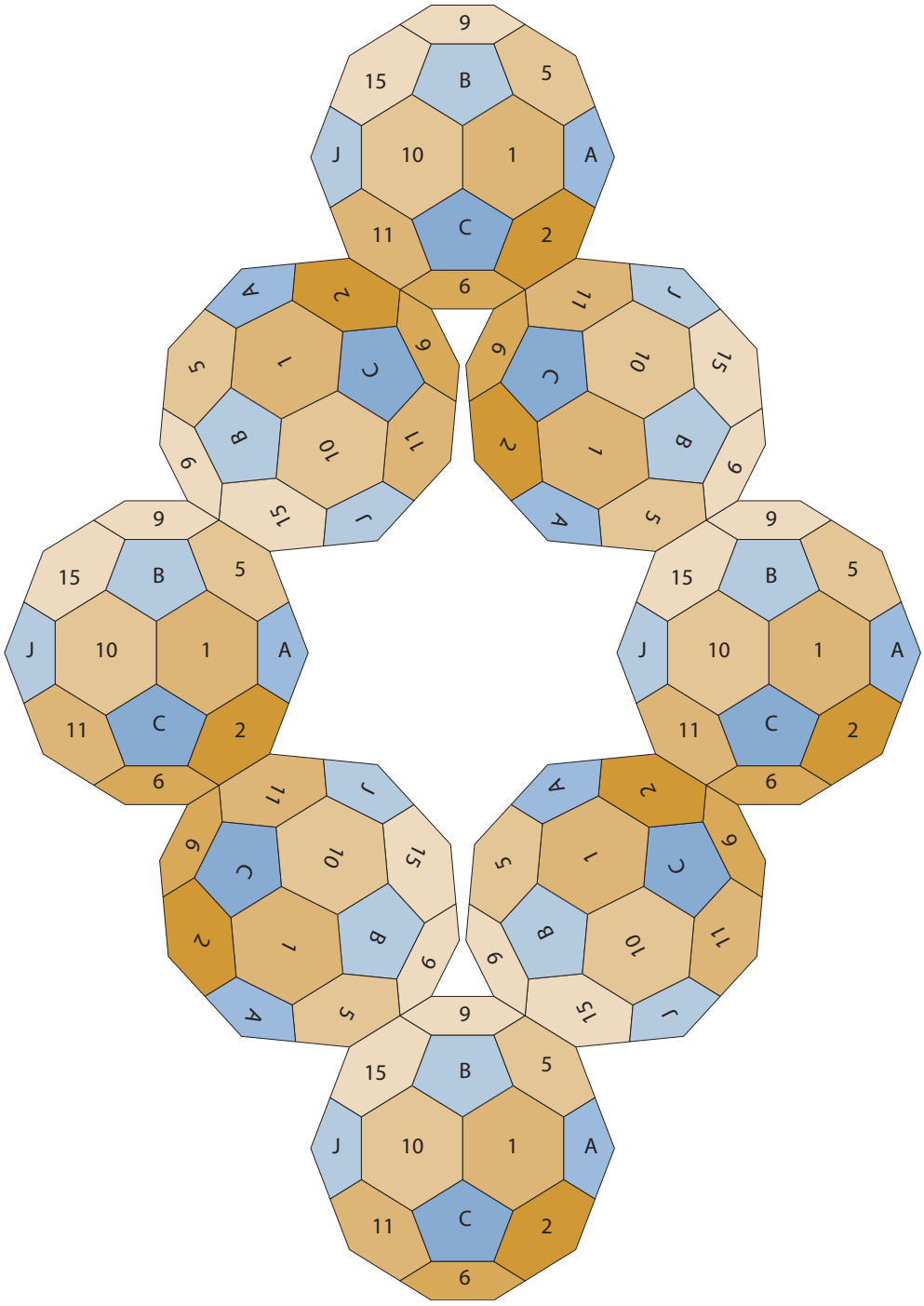


Fig. 24 Triacontahedral face defined by eight icosidodecahedra

The triacontahedral face defined by the eight icosidodecahedra is parallel to the projection plane. For each of its icosidodecahedral units, it is parallel to the diameters between edges 3,4 and16,20 and between edges 6,12 and 9,14 and perpendicular to the diameter between edges 1,10 and13,8.

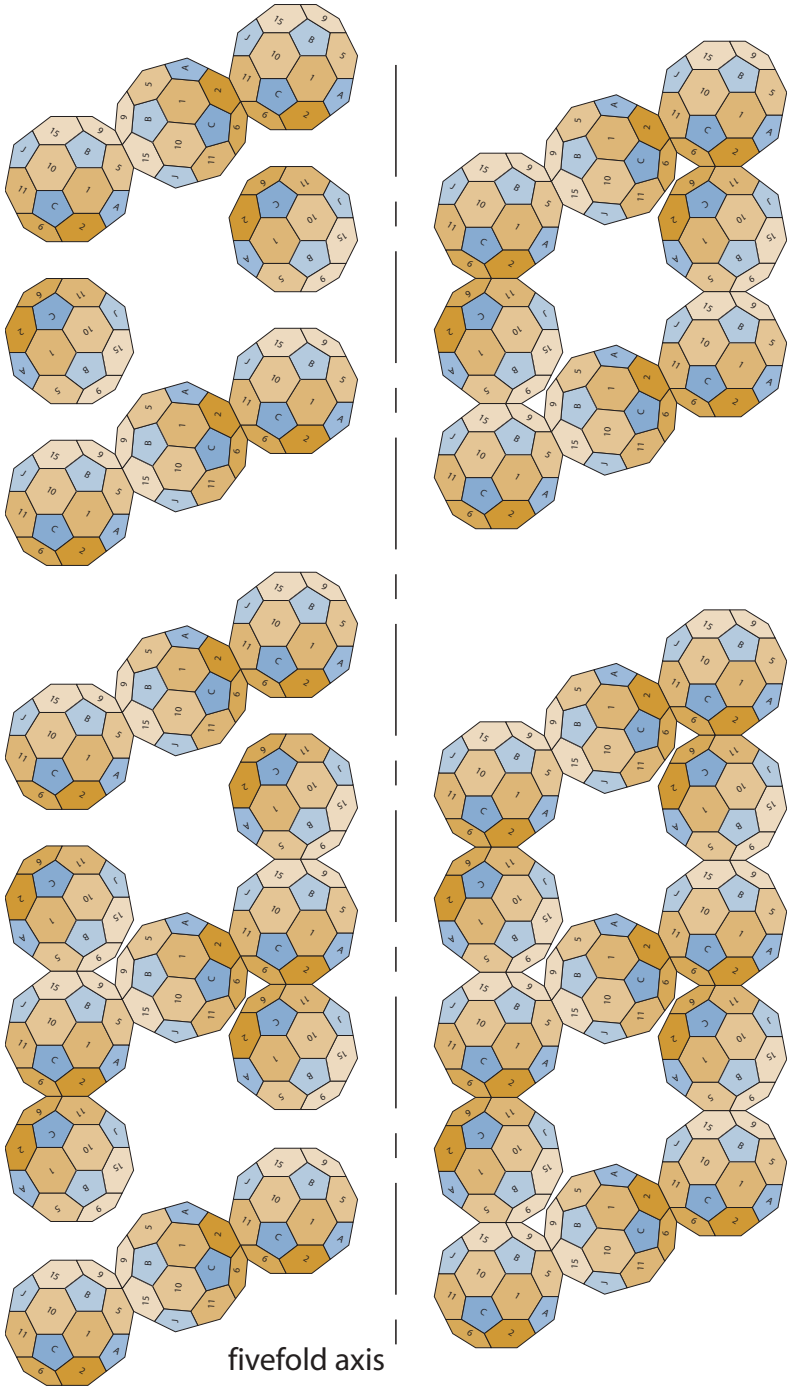


Fig. 25 Triacontahedron–facial assembly and extended facial assembly

The figure shows how a triacontahedron can be lengthened parallel to a fivefold axis. The mid-layer of the triacontahedral assembly consists of ten joiner units. The faces formed by the mid-layer with the upper and lower assemblies is shown on the top. At bottom, the mid-layer consists of forty units and the resulting face is as shown on the right. The elongation could be increased.

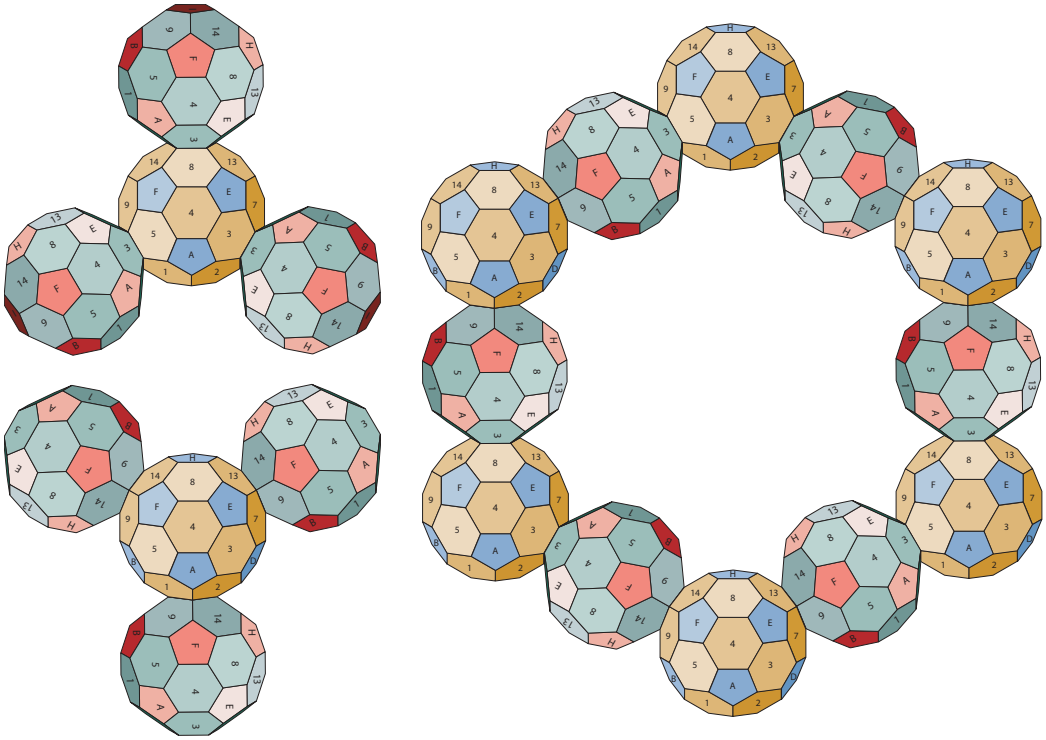


Fig. 26 Ring and hub assemblies–base and α-joiners
The figure shows two a-hubs with a ring composed of six base units and six α-joiners.

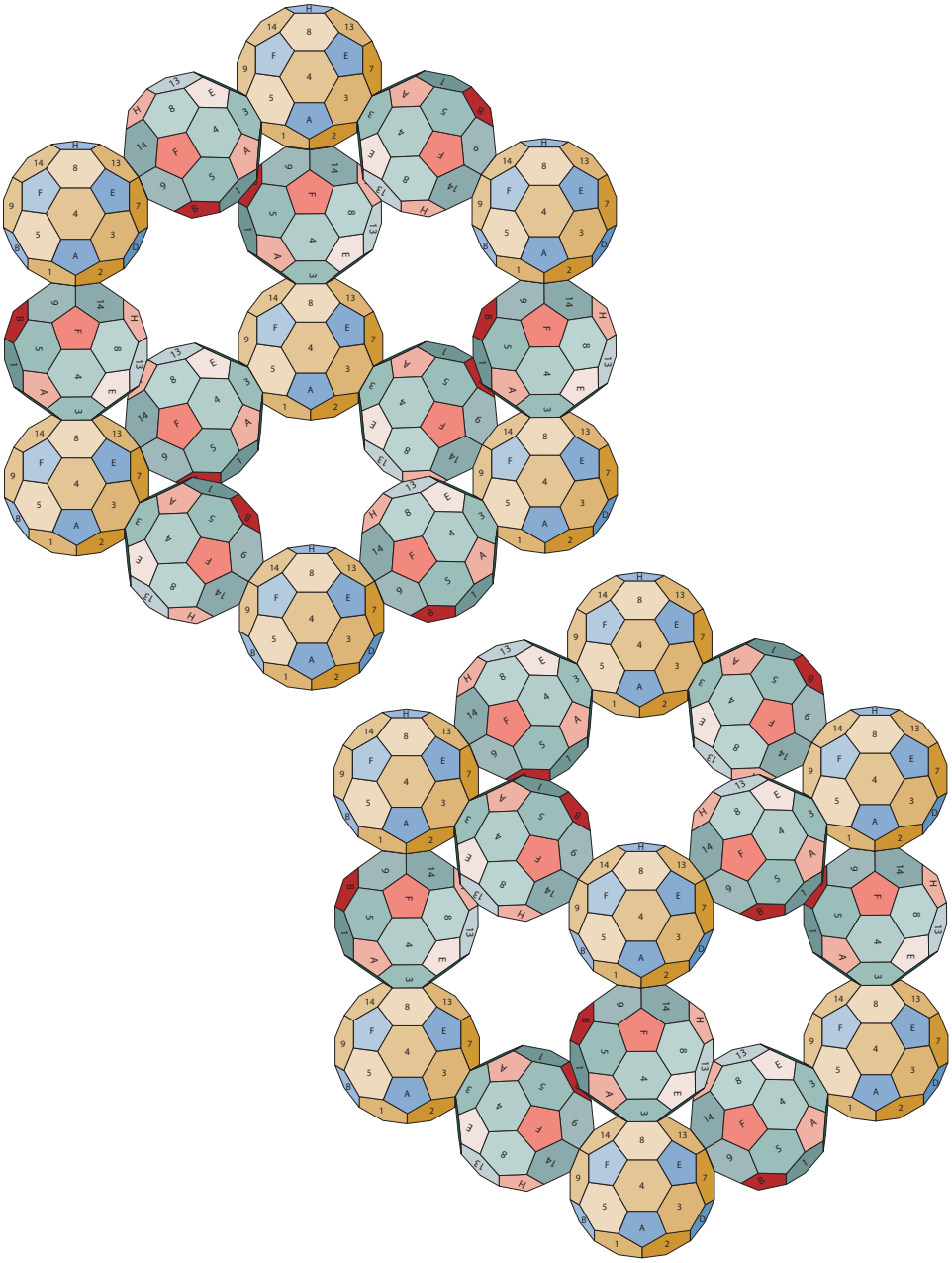


Fig. 27 Triacontahedron–threefold vertexes

The ring and hubs of the previous figure are used here to make threefold vertexes of the triacontahedron. The assembly at the top is concave towards the viewer; the assembly at the bottom is convex towards.

Combining the two hubs with a single ring produces an obtuse Penrose rhombohedral assembly.

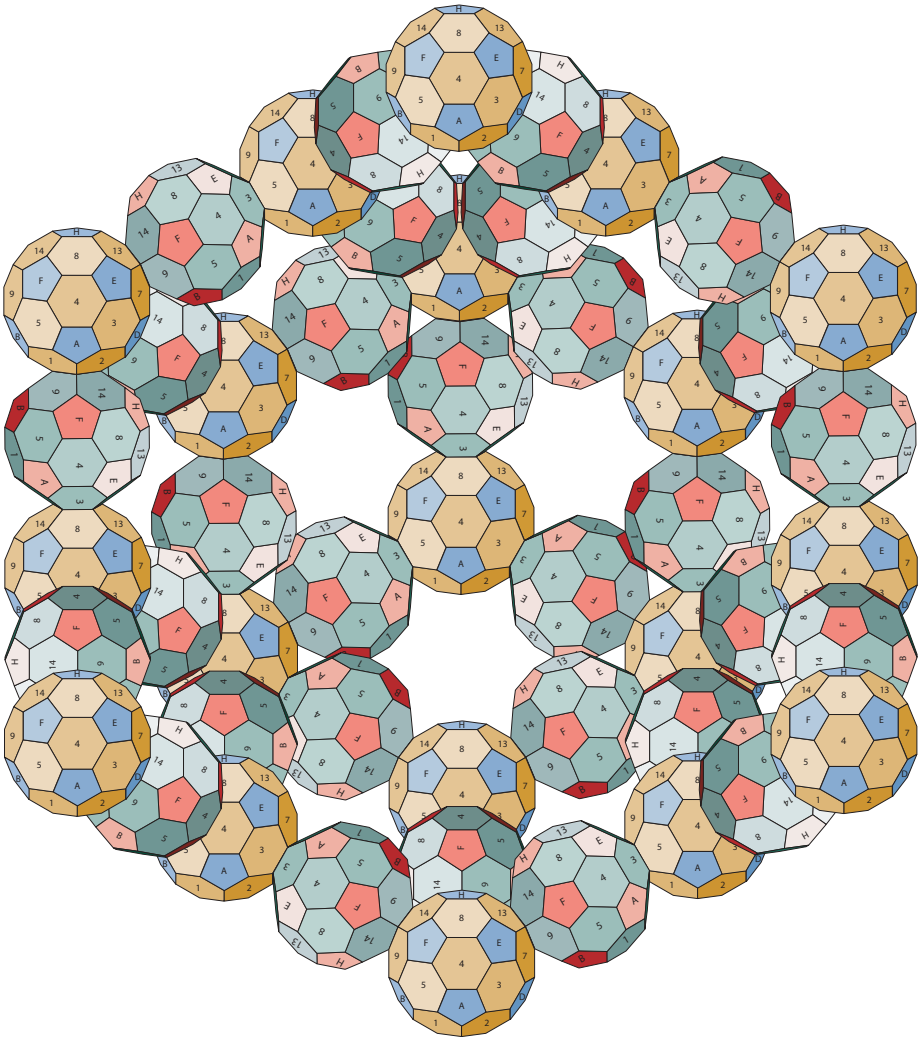


Fig. 28 Triacontahedron–lower portion

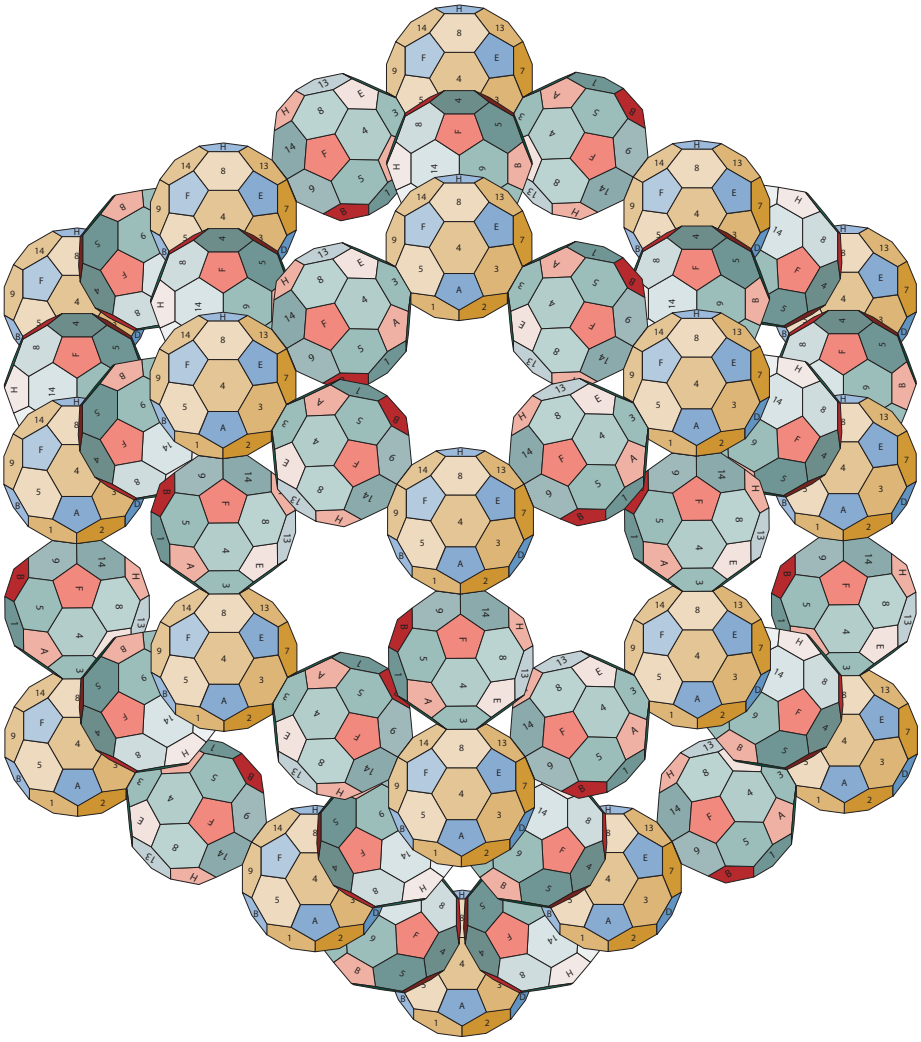


Fig. 29 Triacontahedron–upper portion

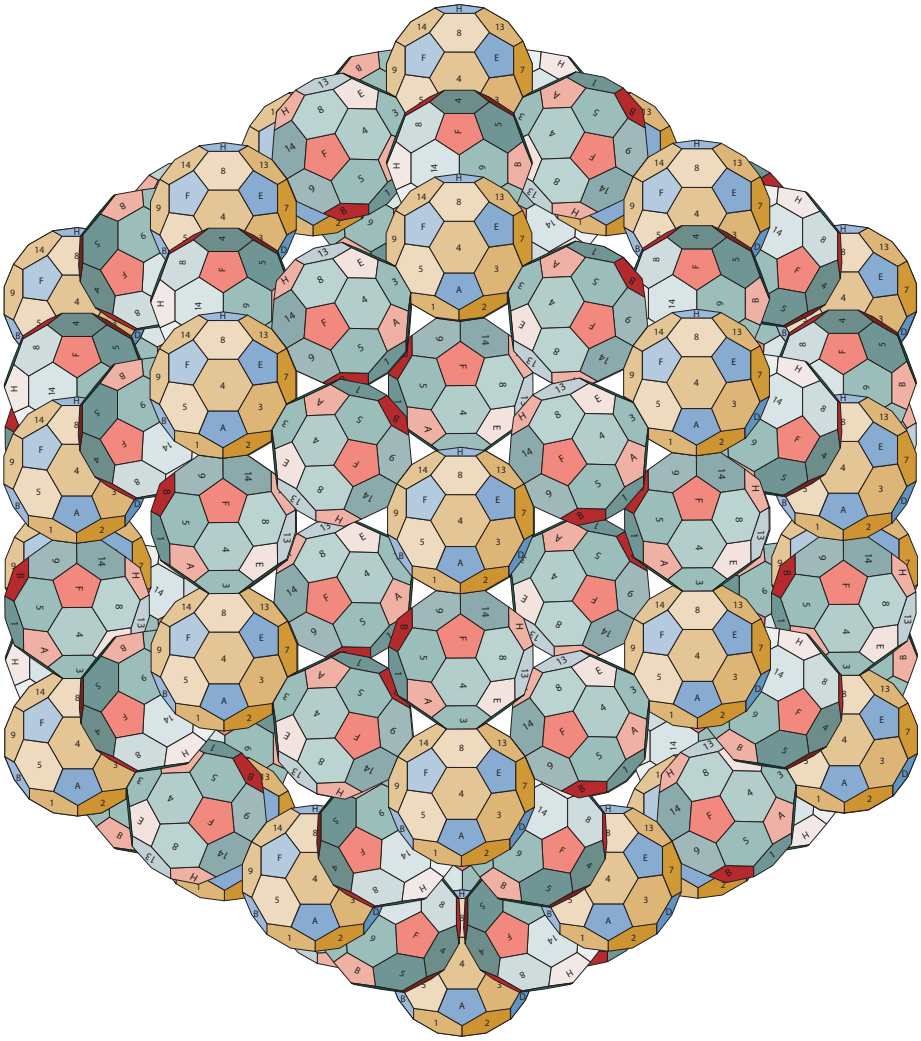


Fig. 30 Triacontahedron–complete

Elongated triacontahedron

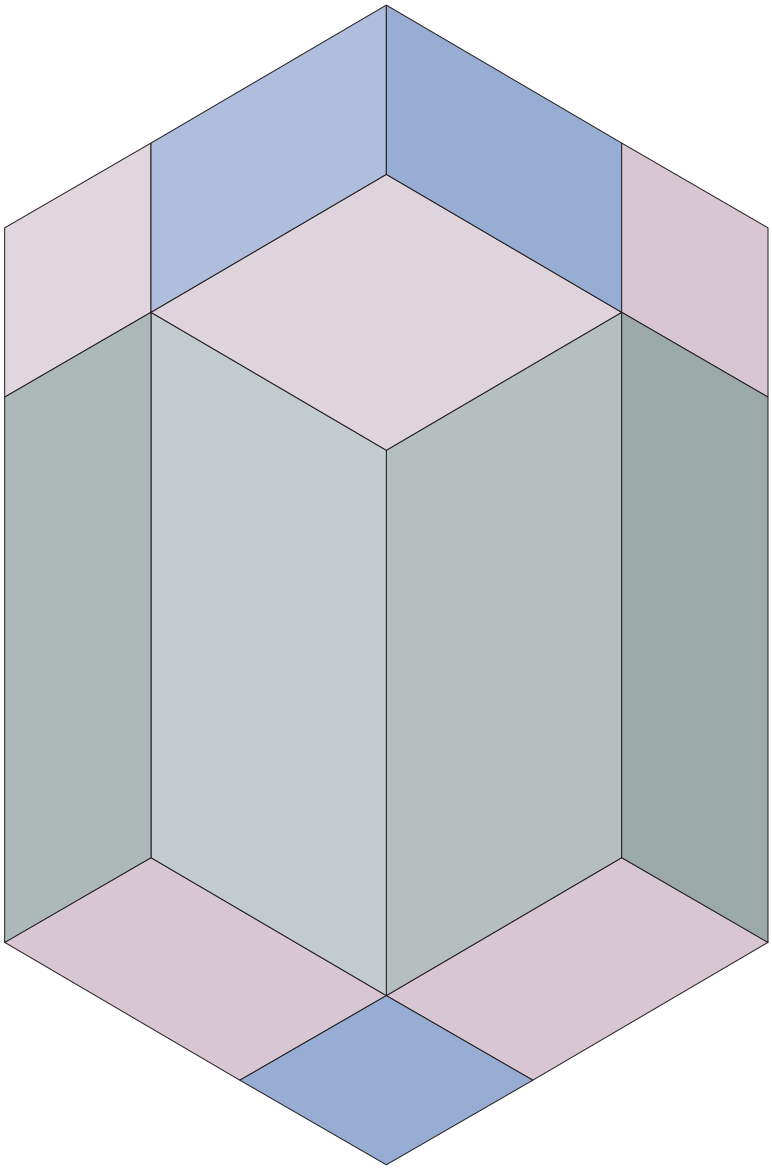


Fig. 31 Elongated triacontahedron viewed parallel to six faces

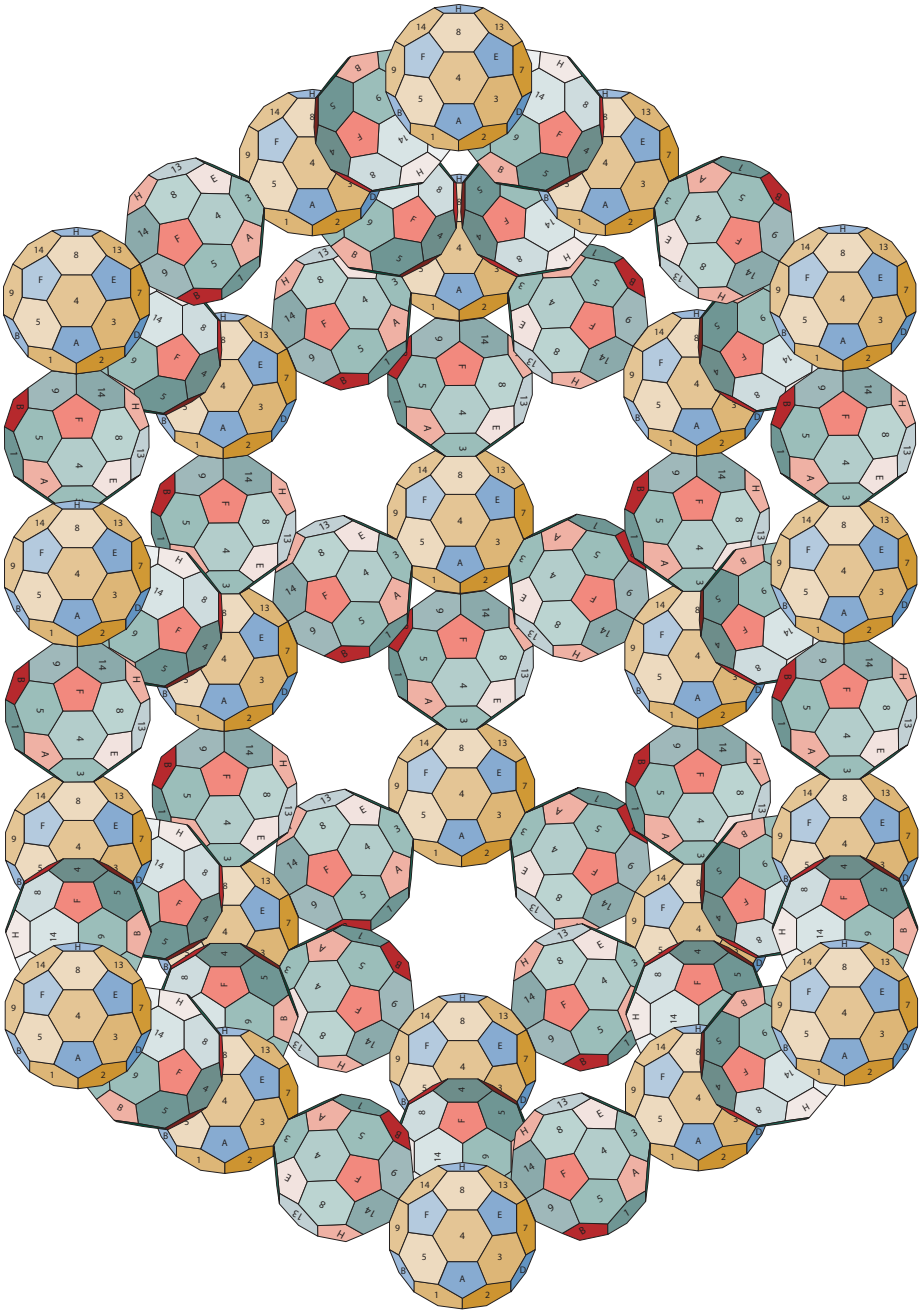


Fig. 32 Elongated triacontahedron–lower portion

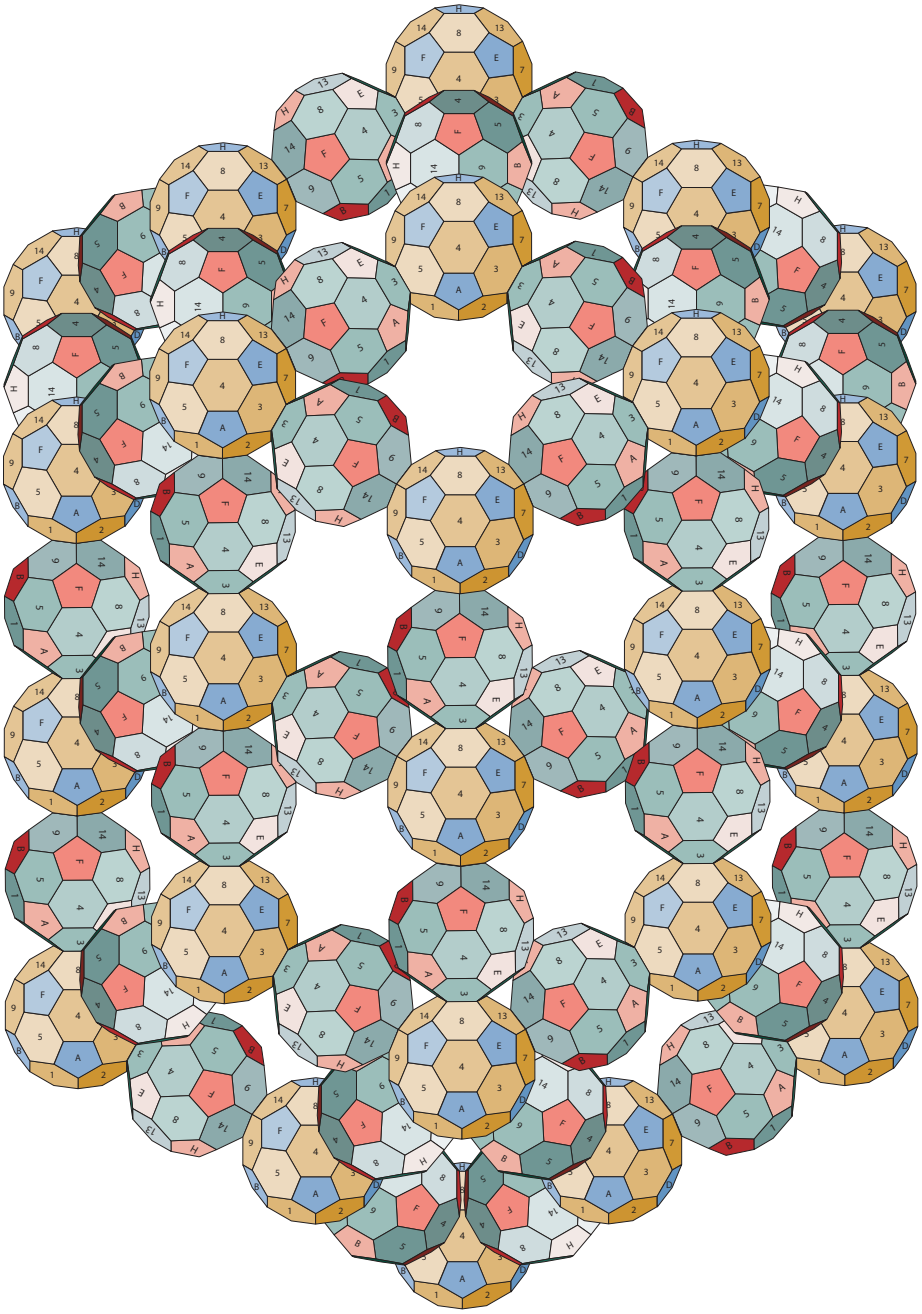


Fig. 33 Elongated triacontahedron–upper portion

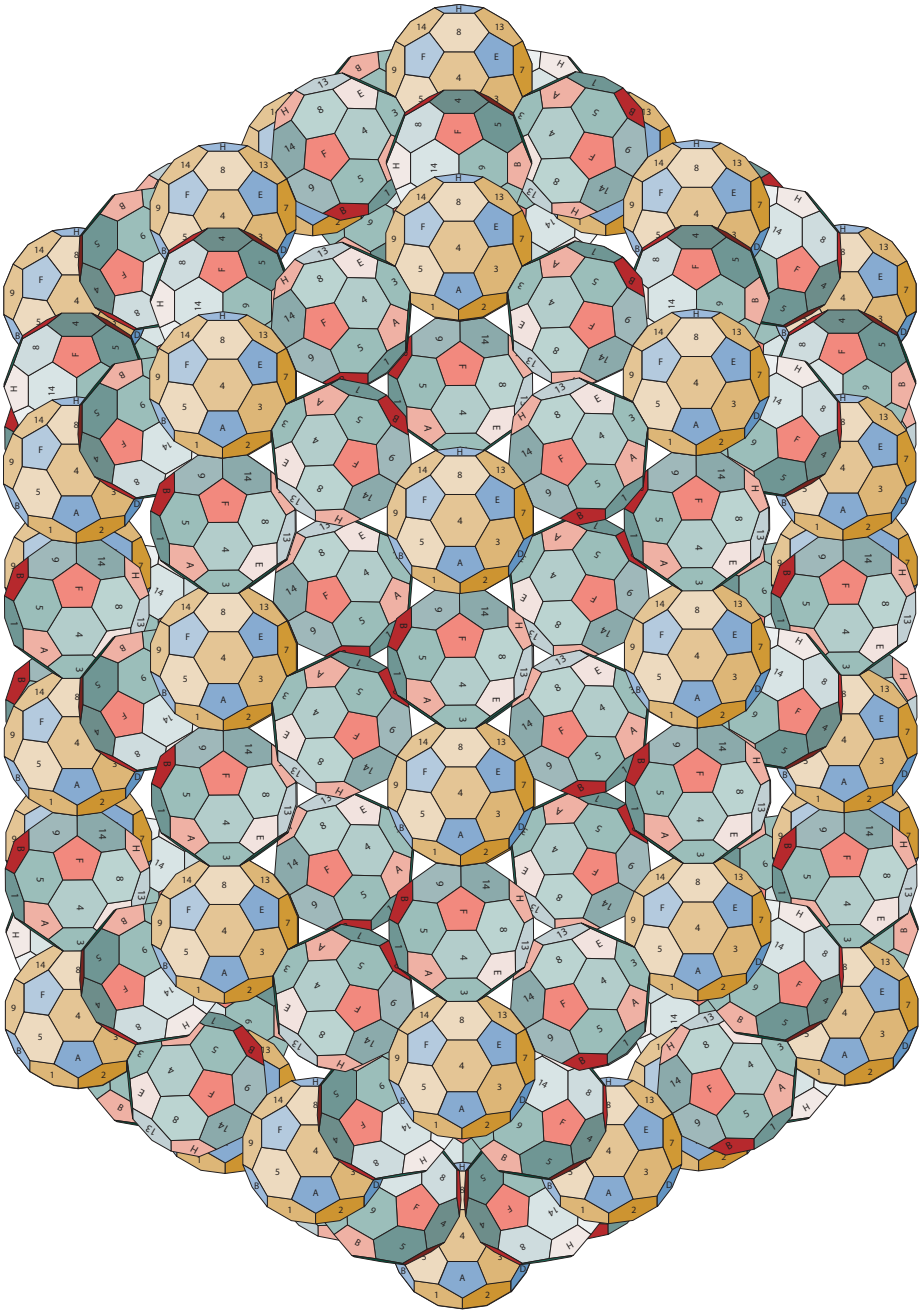


Fig. 34 Elongated triacontahedron–complete

Icosahedron

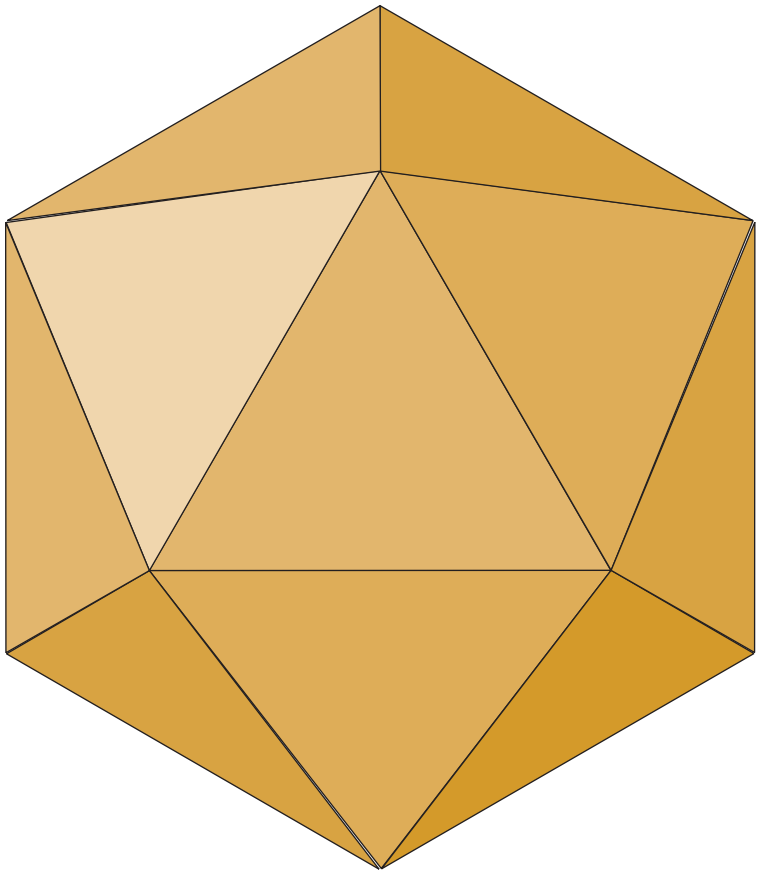


Fig. 35 Icosahedron–facial view

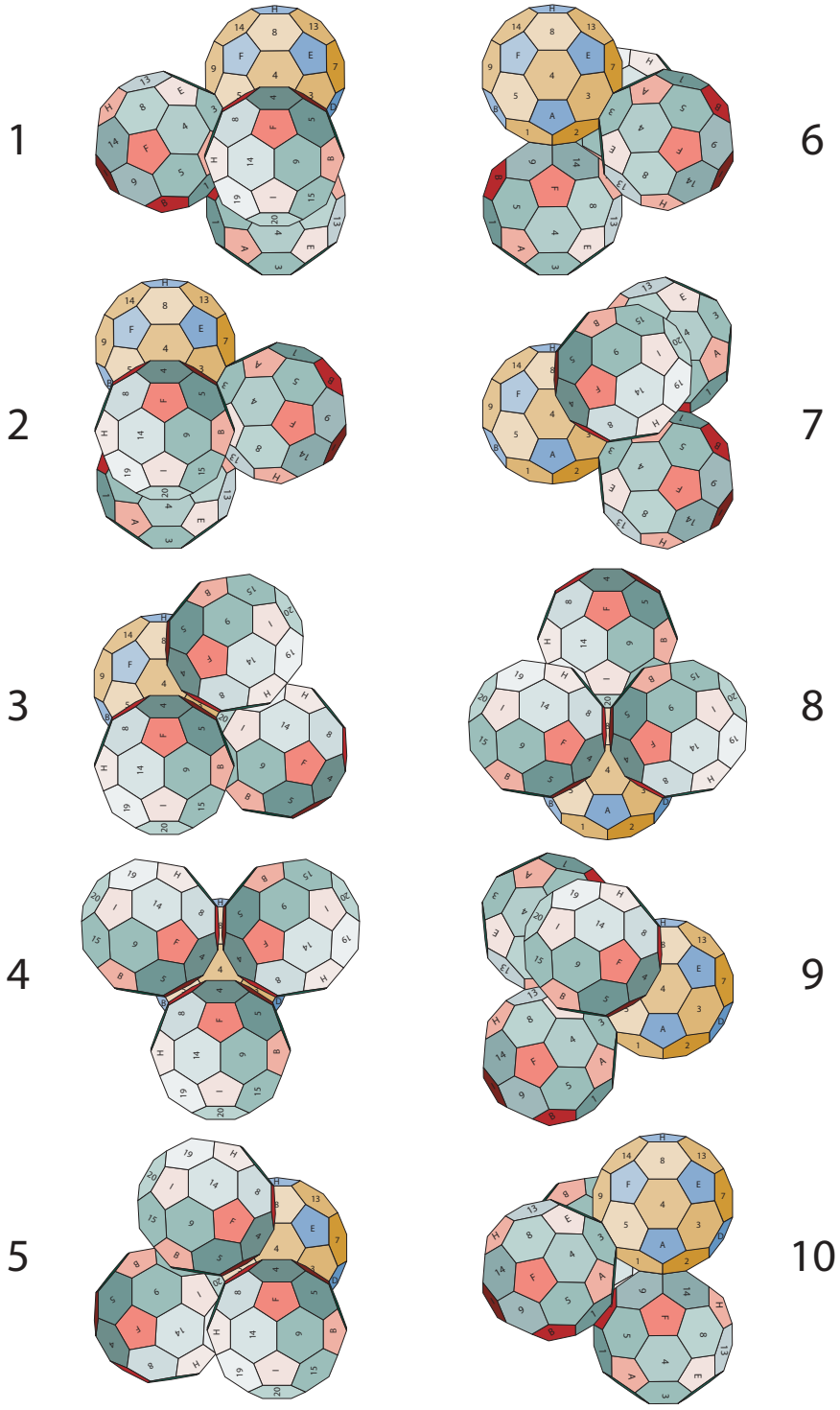


Fig. 36 γ-hubs 1 through 10

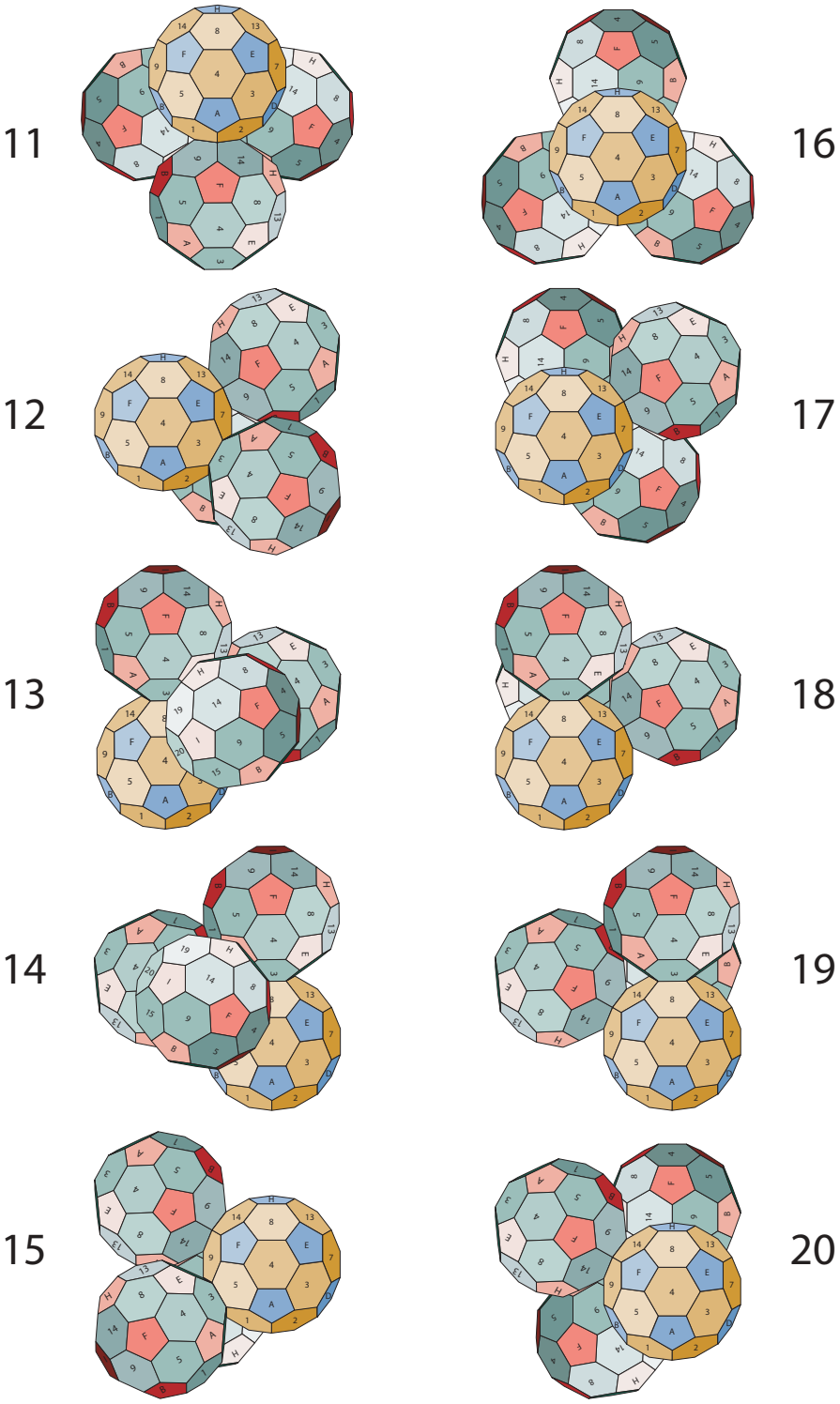


Fig. 37 γ-hubs 11 through 20

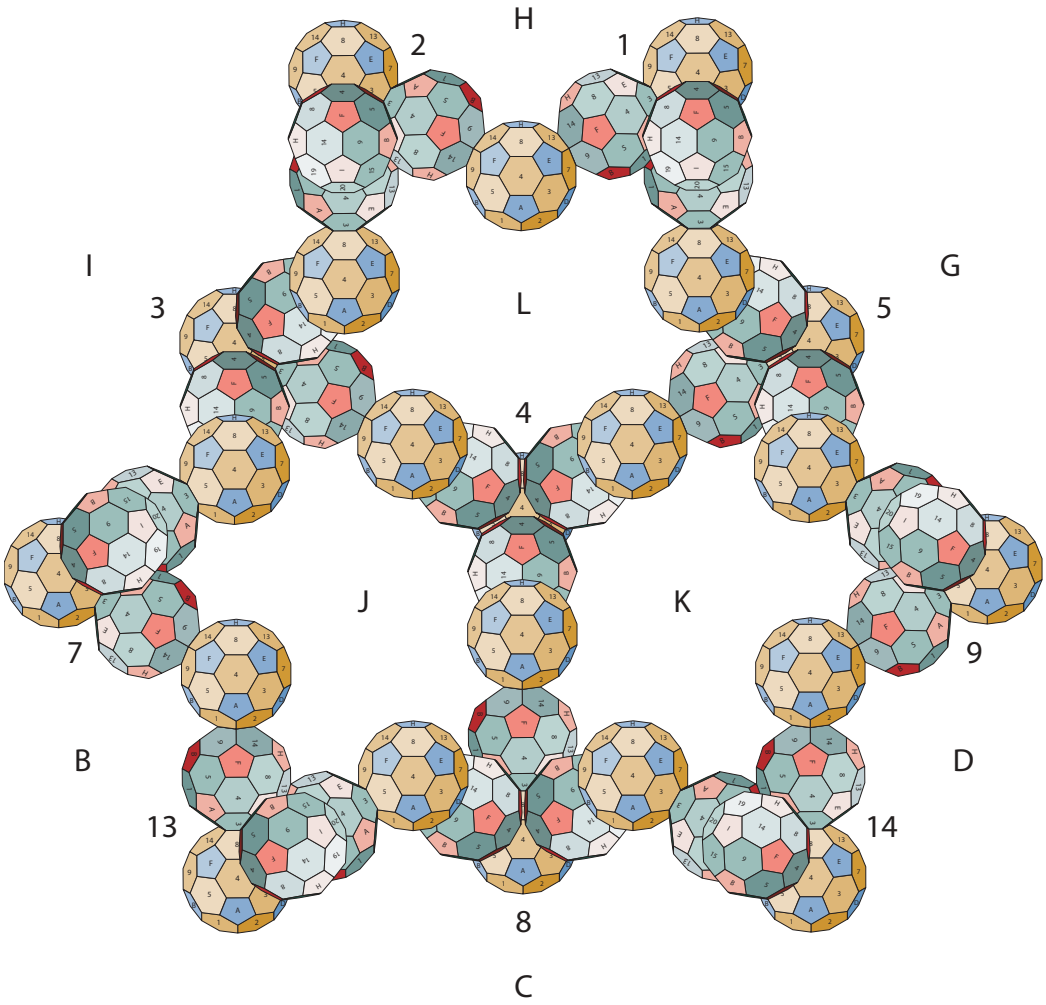


Fig. 38 Icosahedral assembly of γ-hubs–lower portion

The figure shows ten hubs joined by base units. The axis of each hub is perpendicular to an icosahedral face. The ten hub assembly defines each of the lower faces of a facially viewed icosahedron.Hub 4 is connected to hubs 3, 5, and 8. These are connected to hubs 7 and 2, 1 and 9, and 14 and 13.

Base units are used between each pair of hubs. There are three connecting 4 to 3, 5, and 8, two connecting 3 to 2 and 7, 5 to 1 and 9, and 8 to 14 and 13, one connecting 1 to 2, 9 to 14, and 7 to 13. The angled hub to base to hub connection defines a plane which is perpendicular to an edge of the icosahedron.

Hubs 4, 3, 2, 1, and 5 and their connections form a pentagonal ring whose axis of symmetry is normal to face L at its centroid. The same is true for 4, 5, 9, 14, and 8 relative to face K, and 4, 8, 13, 7, and 3 relative to face J. The ring is viewed normal to its plane in Figures 71 and 72.

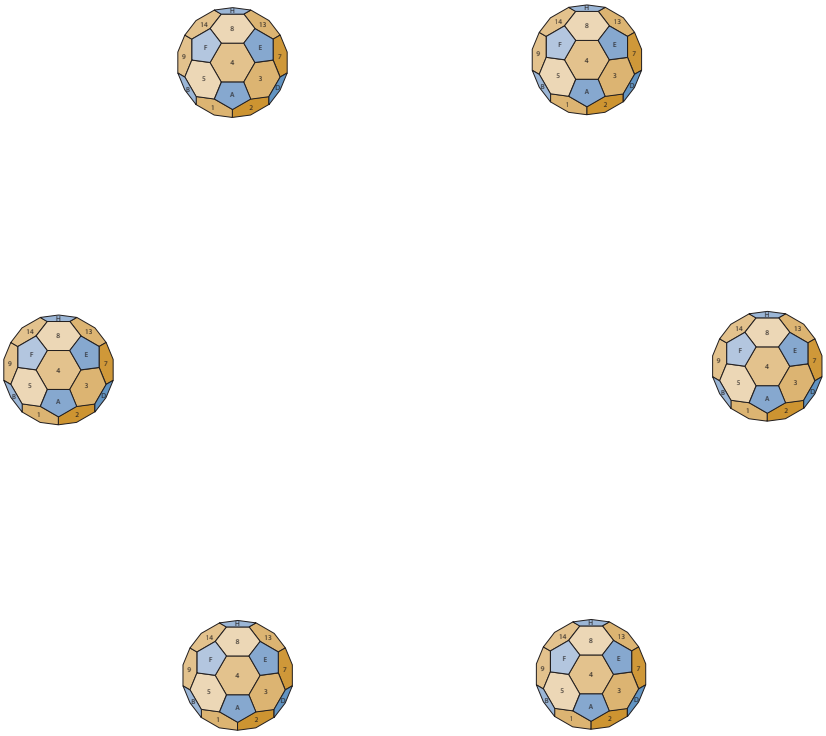


Fig. 39 Icosahedral assembly of γ-hubs–mid-layer
The mid-layer consists of just six base units. The base units lie on the same plane.

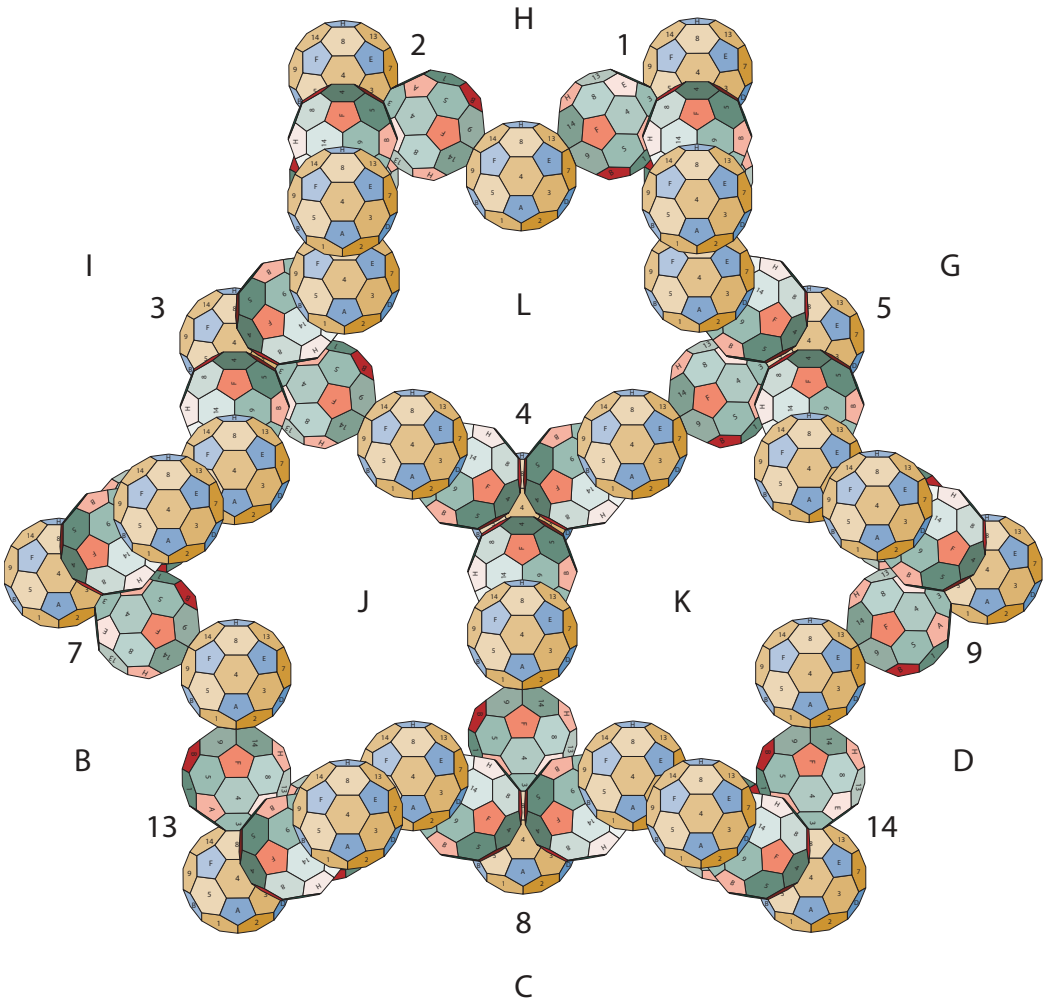


Fig. 40 Icosahedral assembly of γ-hubs–lower portion with mid-layer
The figure shows the lower portion of the icosahedral assembly with the six base units of the mid-layer in place.

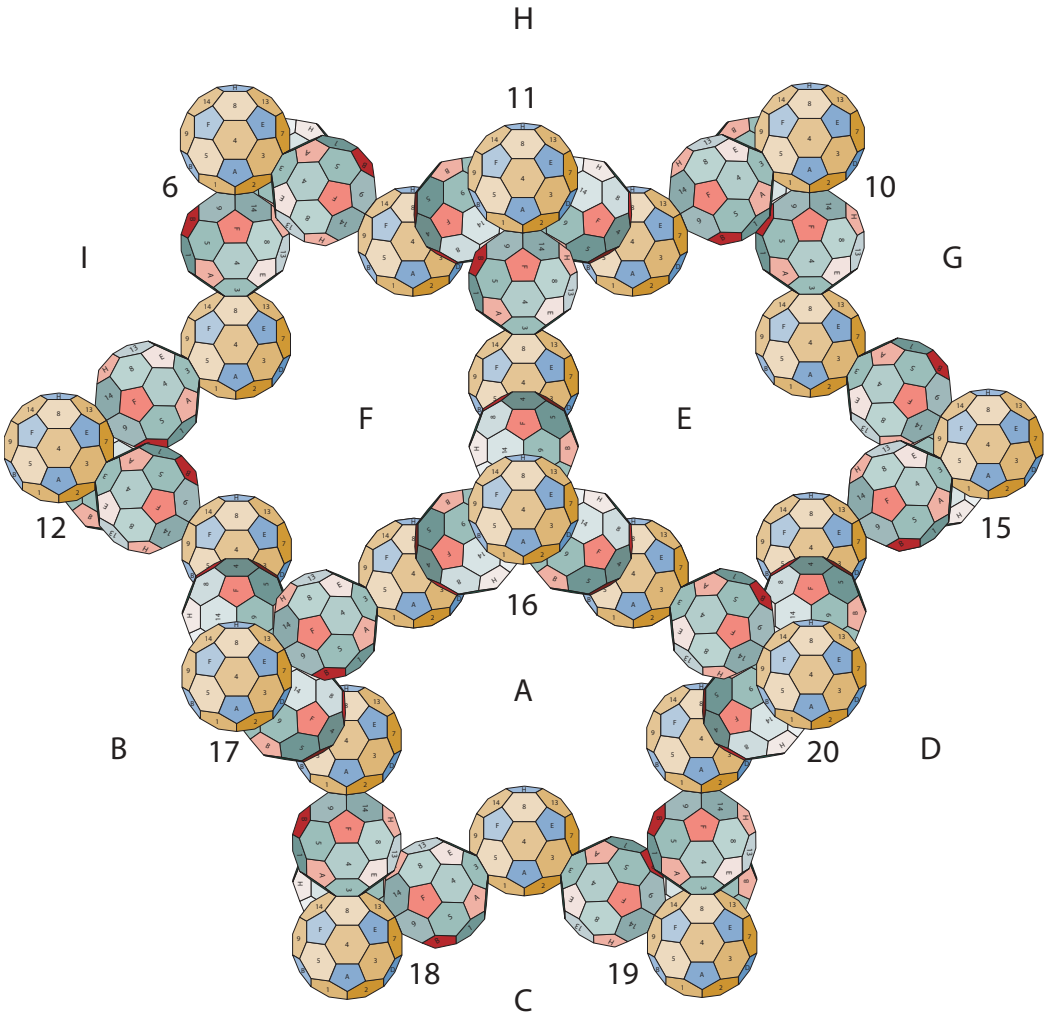


Fig. 41 Icosahedral assembly of γ-hubs–upper portion

The figure shows ten hubs joined by base units. The axis of each hub is perpendicular to an icosahedral face. The ten hub assembly defines each of the upper faces of a facially viewed icosahedron.Hub 16 is connected to hubs 11, 17, and 20. These are connected to hubs 6 and 10, 6 and 12, and 18 and 19.

Base units are used between each pair of hubs. There are three connecting 16 to 11, 17, and 20, two connecting 11 to 6 and 10, 17 to 12 and 18, and 20 to 15 and 19, one connecting 10 to 15, 19 to 18, and 12 to 6.

Hubs 16, 17, 12, 6, and 11 and their connections form a pentagonal ring whose axis of symmetry is normal to face F at its centroid. The same is true for 16, 11, 10, 15, and 20 relative to face E, and 16, 20, 19, 18, and 17 relative to face A.

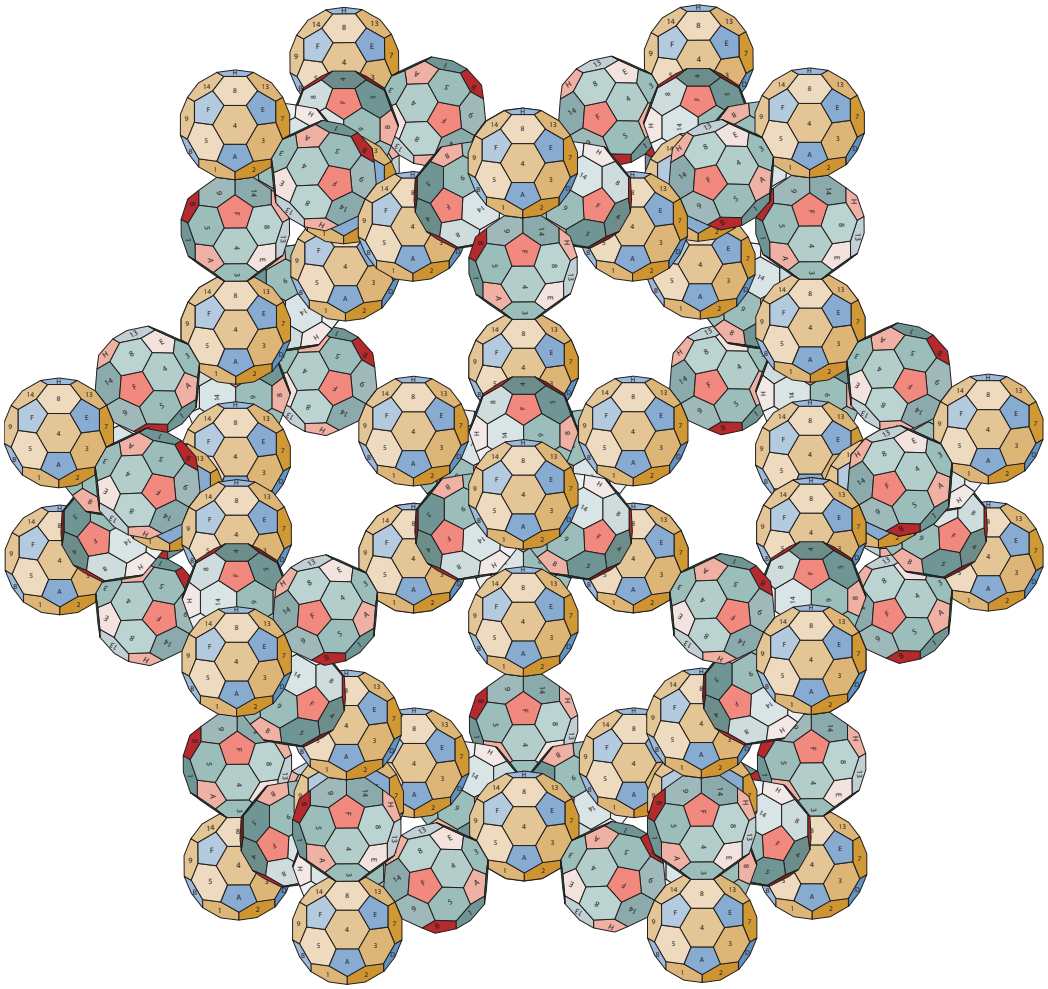


Fig. 42 Icosahedral assembly of γ-hubs–complete

The figure shows the complete icosahedral assembly of twenty hubs and their connectors. An additional six base units join hubs 6 and 2, 10 and 1, 15 and 9, 19 and 14, 18 and 13, and 7 and 12.

Six additional pentagonal rings are formed by the joining of the upper and lower portions of the assembly–2, 3, 7, 12, and 6 whose axis of symmetry is normal to face D at its centroid., 1, 2, 6, 11, and 10 relative to C, 9, 5, 1, 10, and 15 relative to B, 14, 9, 15, 20, and 19 relative to I, 13, 8, 14, 19, and 18 relative to H, and 7, 13, 18, 17, and 12 relative to G.

Pentagonal view

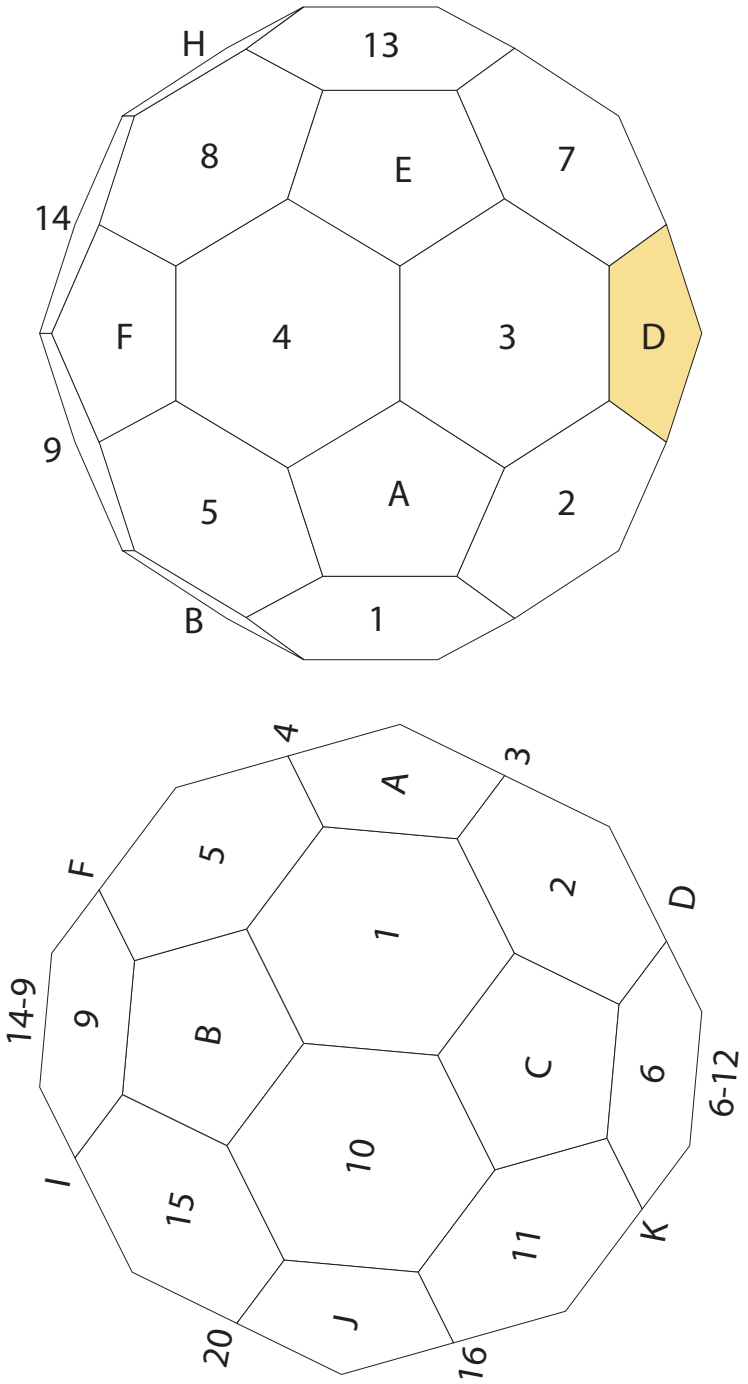


Fig. 43 Joiner unit for base unit rotation β

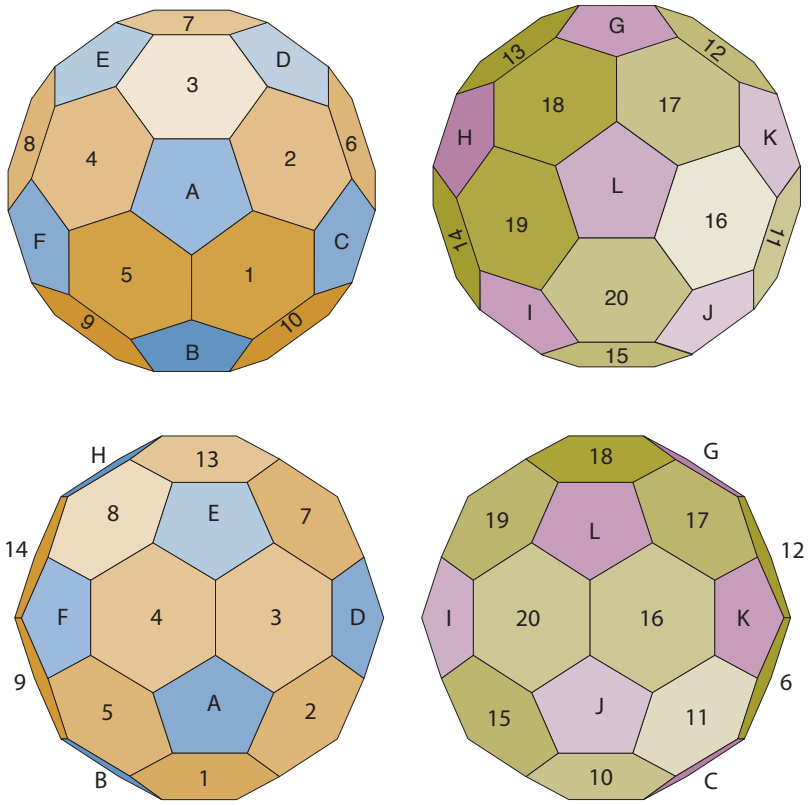


Fig. 44 Pentagonal views of base unit and β-joiner

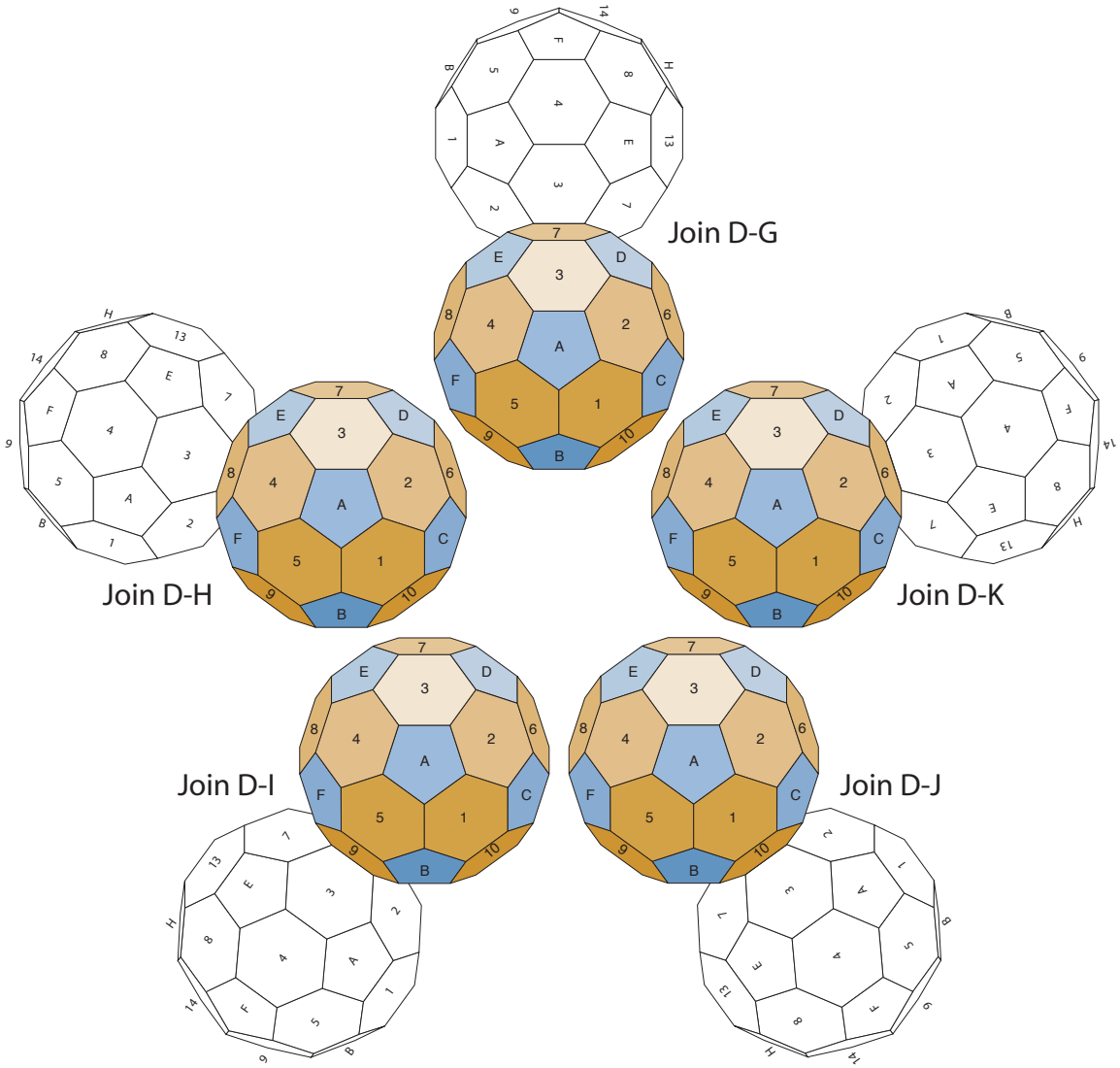


Fig. 45 Base unit joins with β-joiner face D

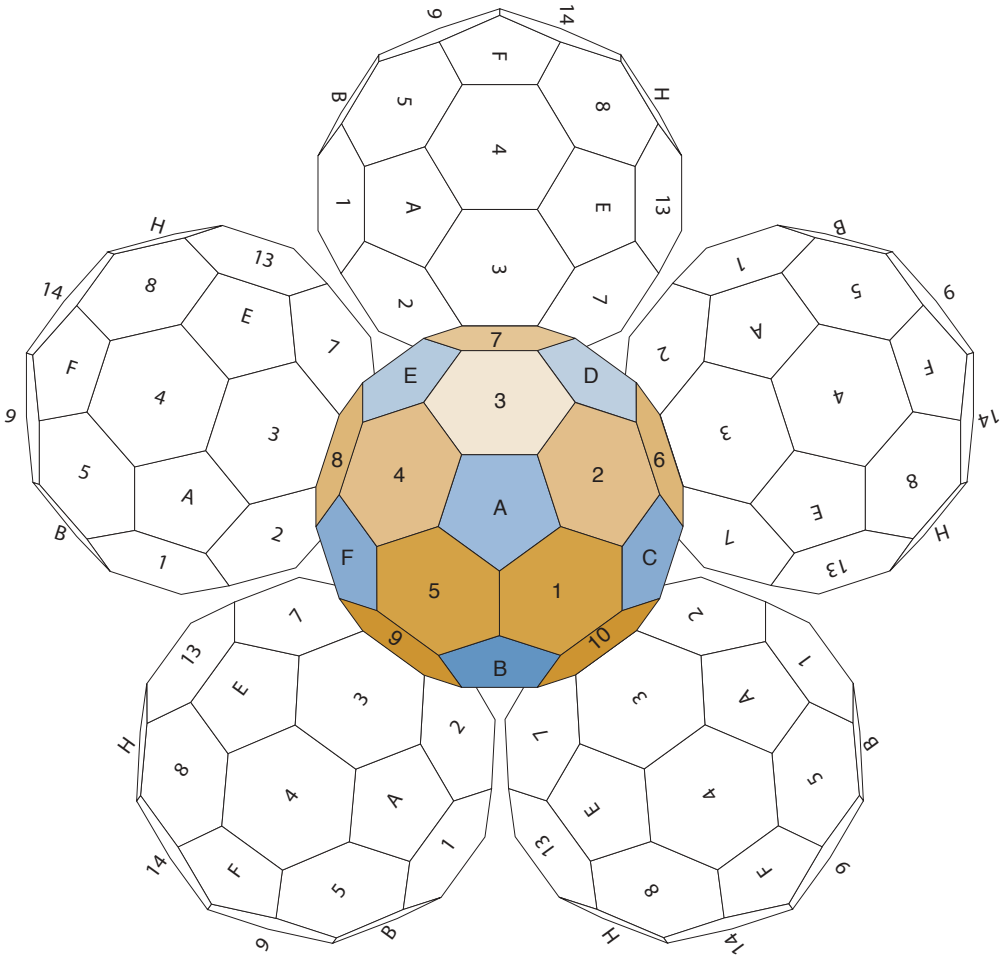


Fig. 46 β-hub L

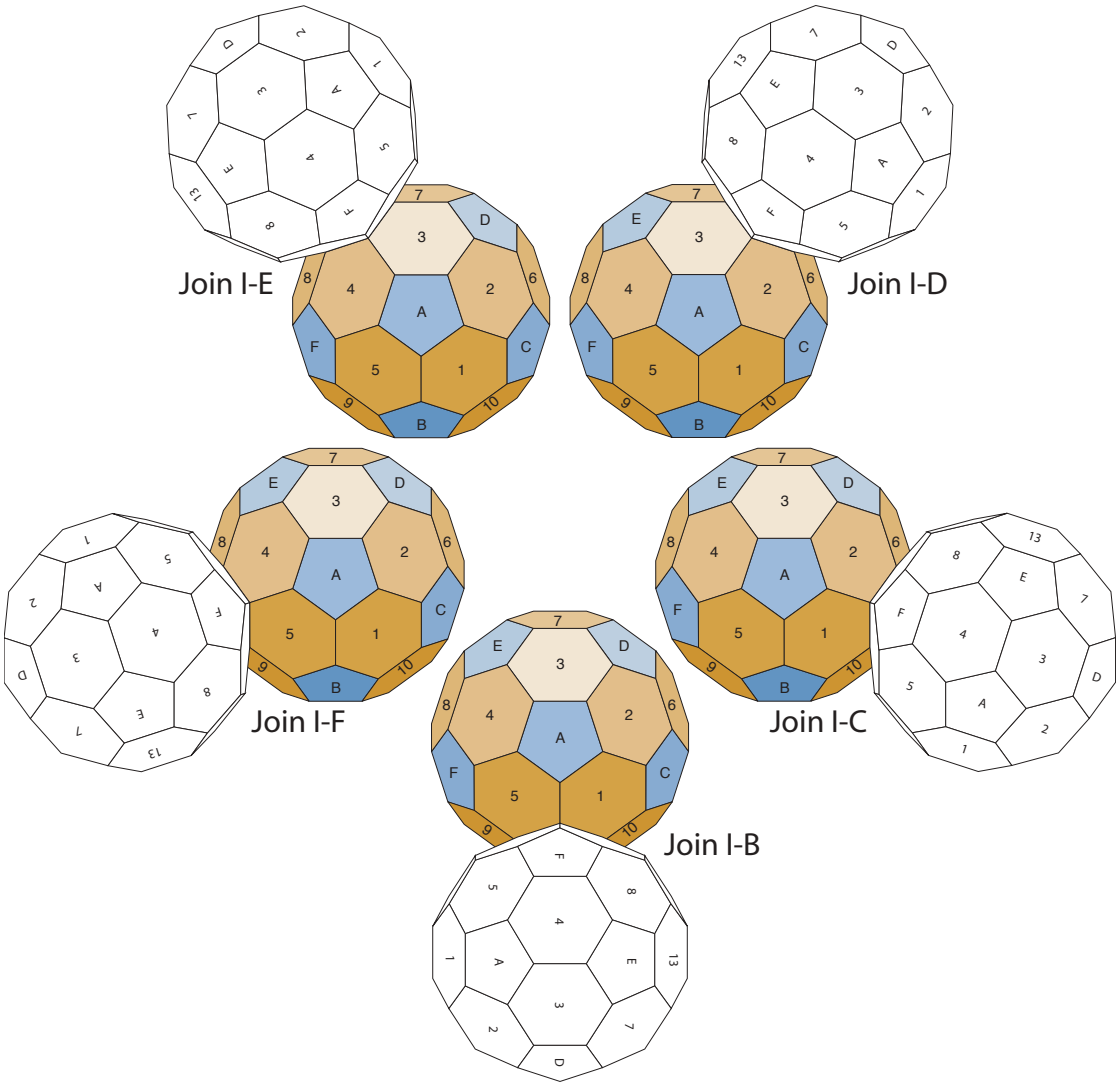


Fig. 47 Base unit joins with β-joiner face I

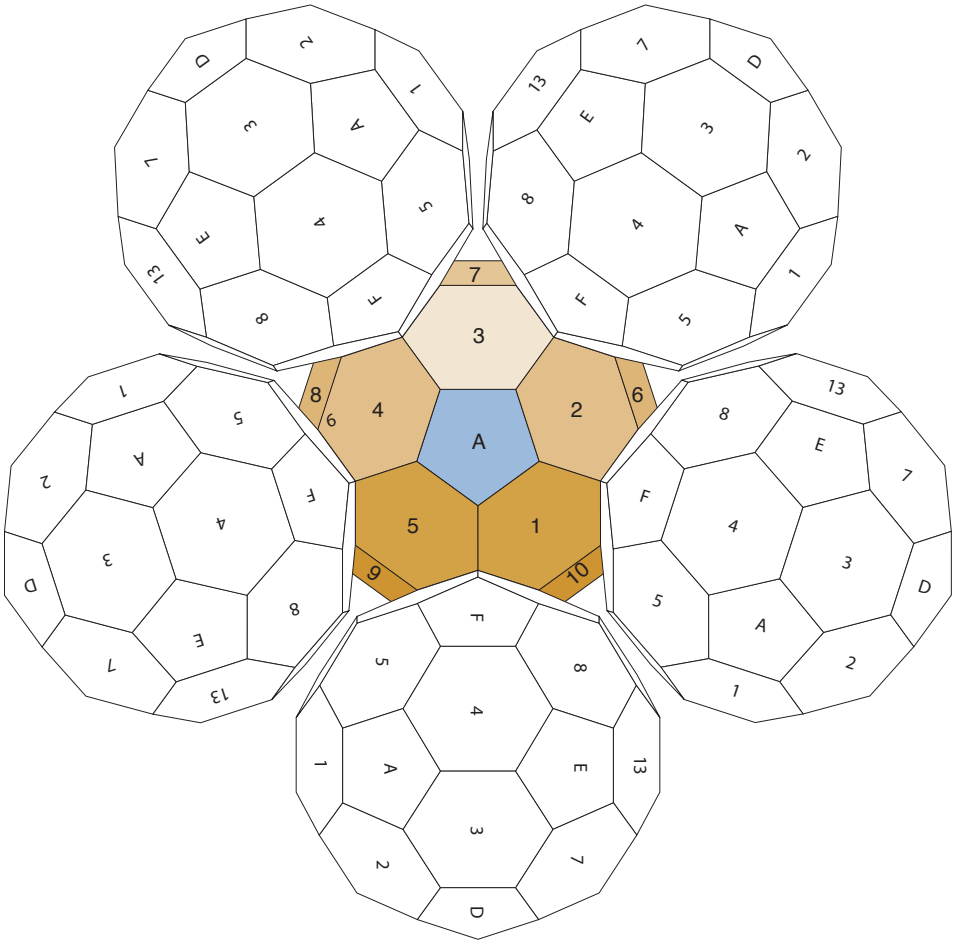


Fig. 48 β-hub A

Pentagonal dodecahedron

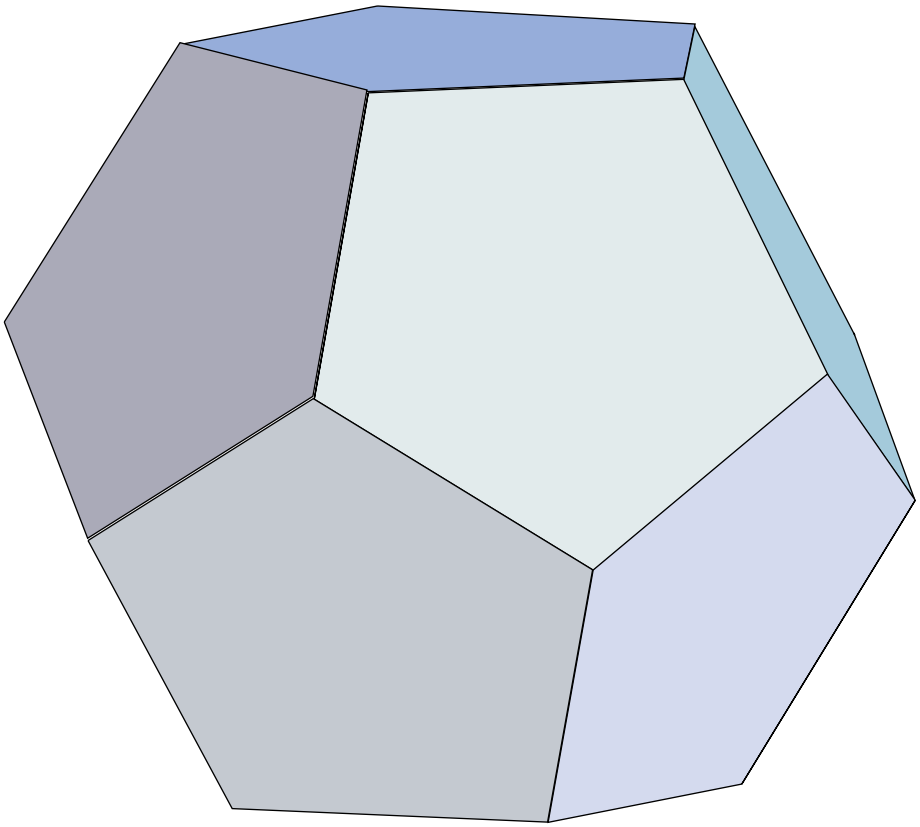


Fig. 49 Pentagonal dodecahedron

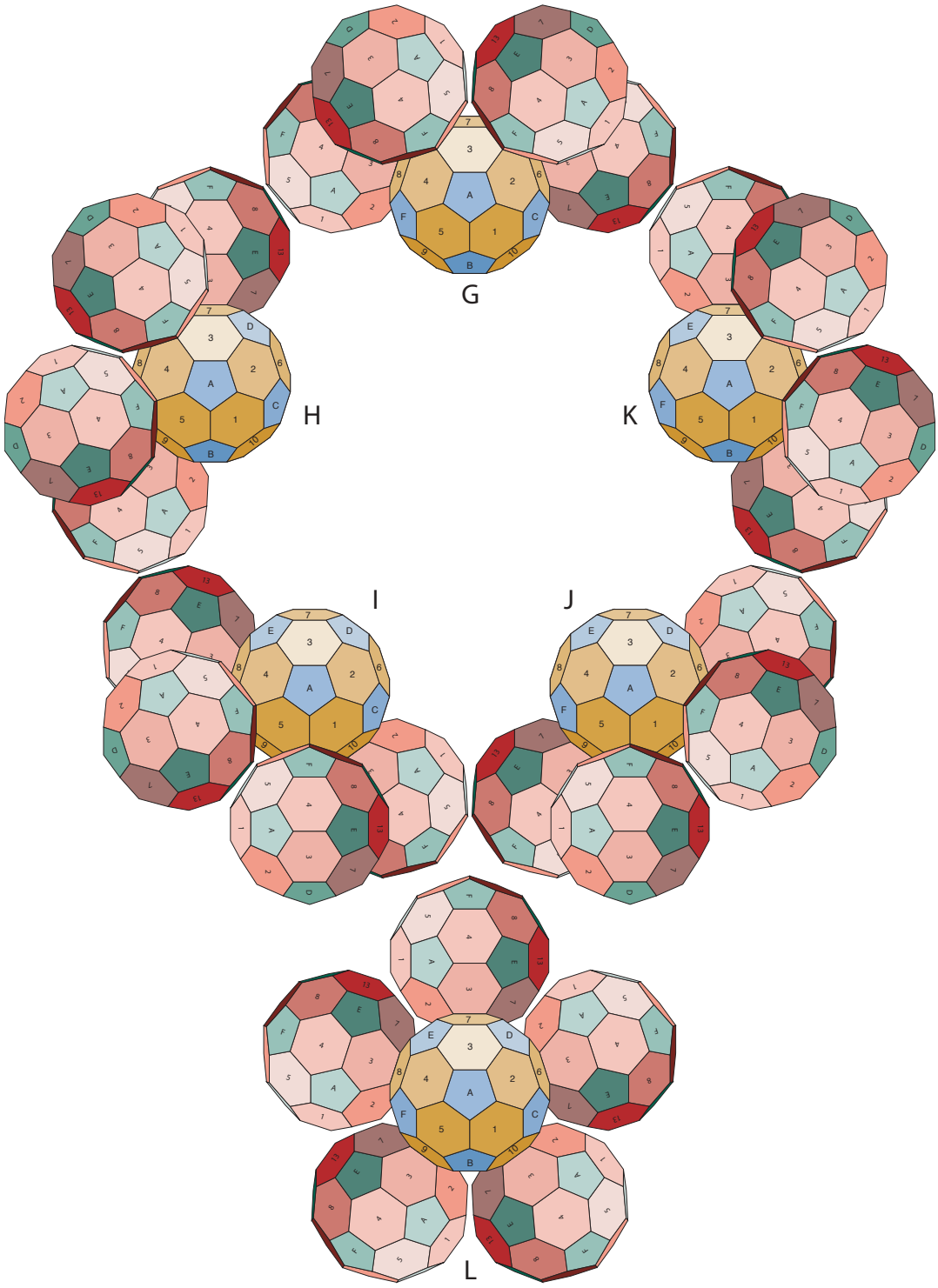


Fig. 50 β-hubs G through L

The figure shows the six β-hubs of the lower assembly of the pentagonal dodecahedron.

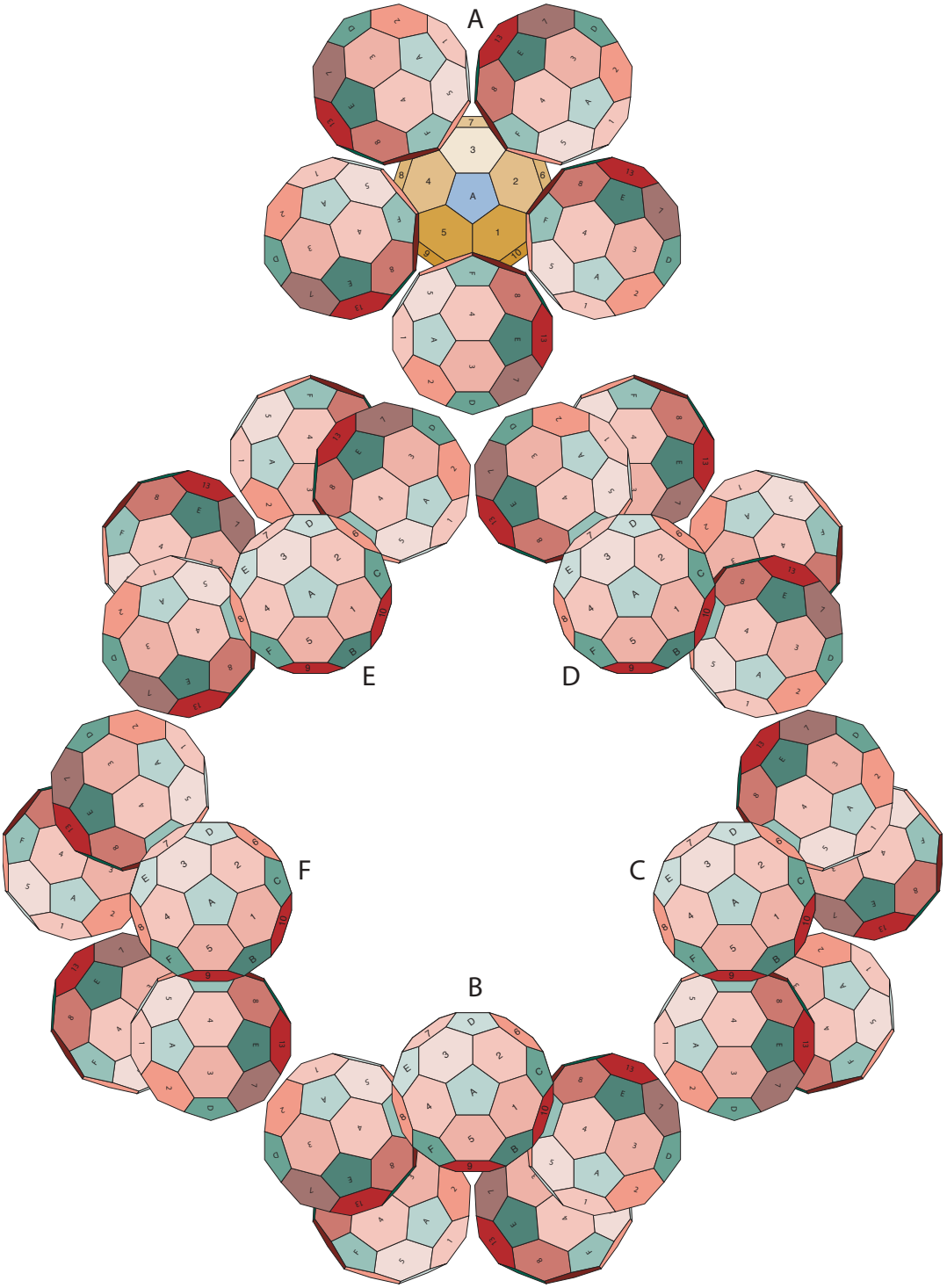


Fig. 51 β-hubs A through F
The figure shows each of the six hubs of the upper assembly.

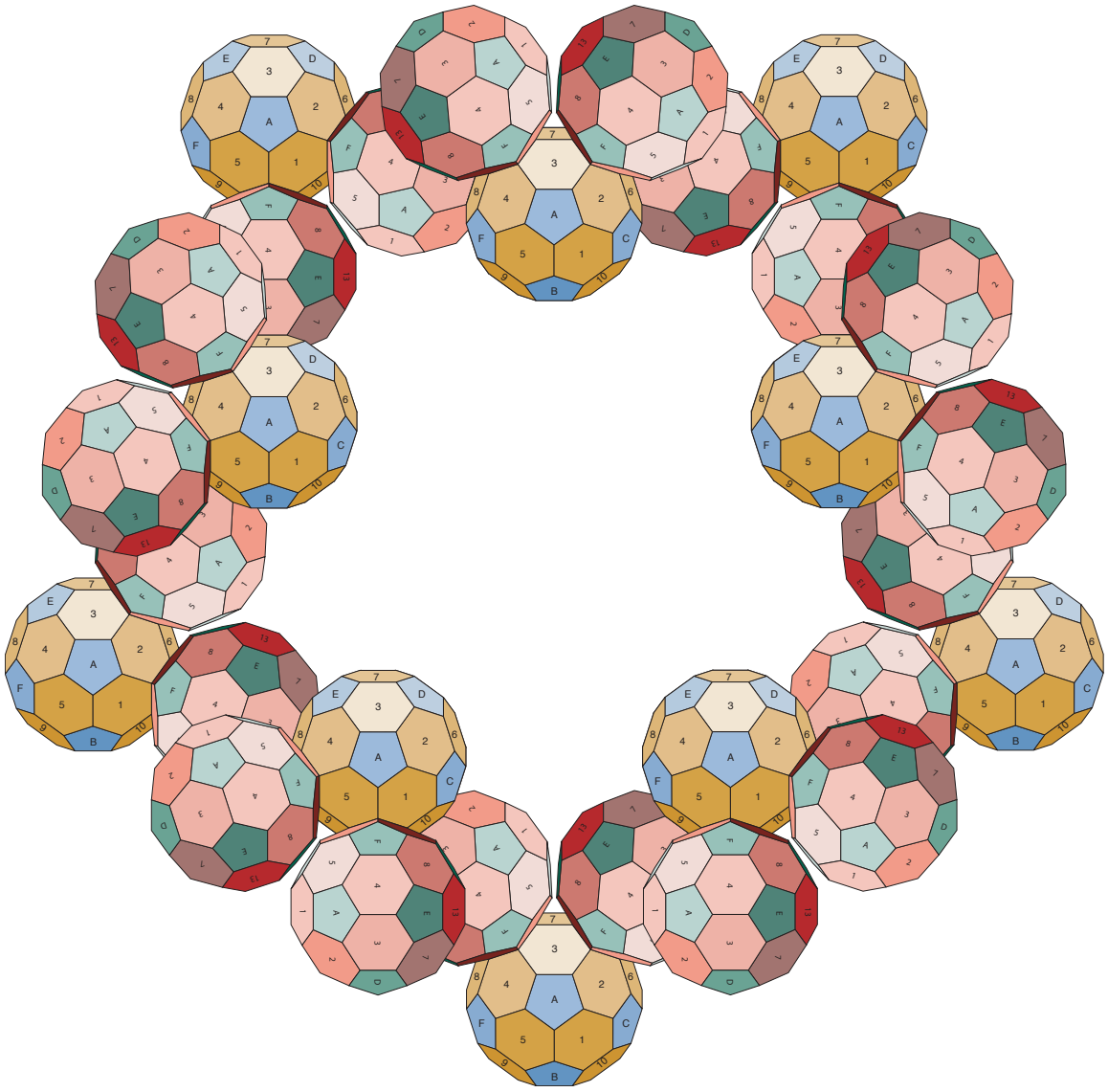


Fig. 52 Pentagonal dodecahedral assembly–lower mid-layer

The figure shows the assembly of five of the β-hubs of the lower assembly–G, H, I, J, and K. Each of the five hubs is connected to each of the two adjacent hubs by a base unit to form a structural coherent ring.

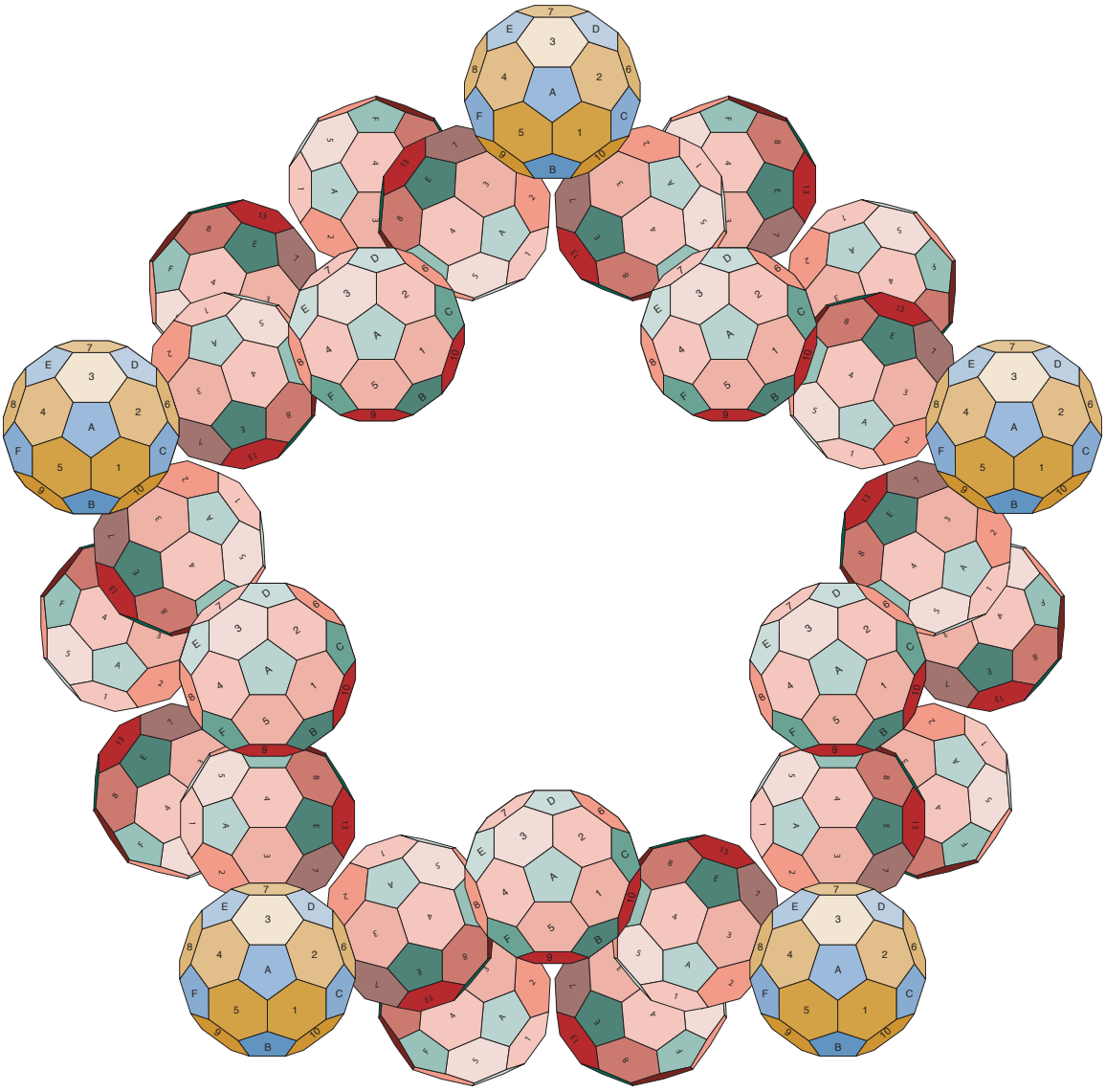


Fig. 53 Pentagonal dodecahedral assembly–upper mid-layer
The figure shows five hubs of the upper assembly –B, C, D, E, and F–joined by base units to form a structurally coherent ring.

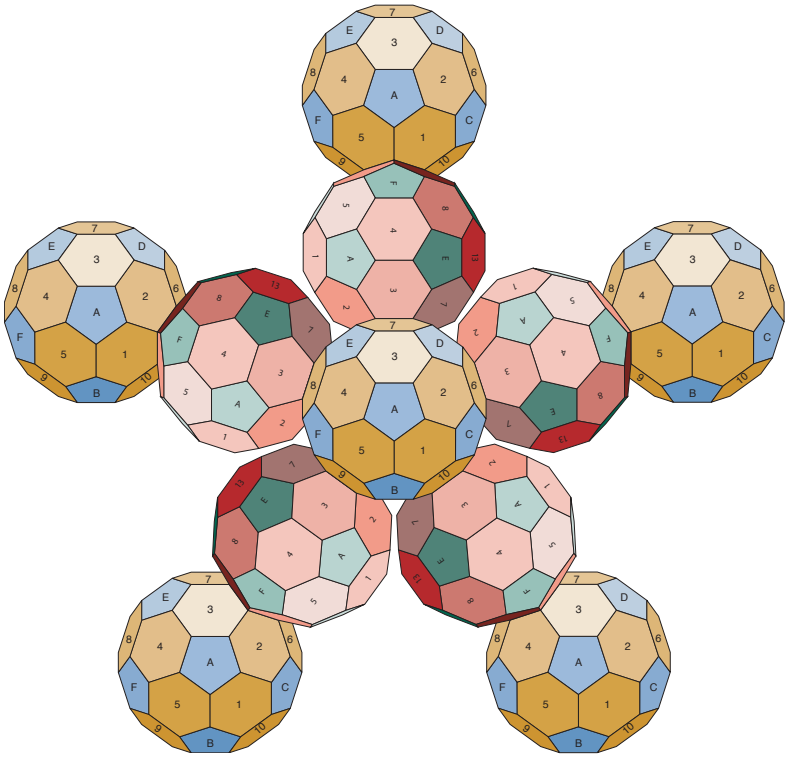


Fig. 54 Pentagonal dodecahedral assembly–bottom hub
The figure shows hub L of the lower assembly with five base units attached.

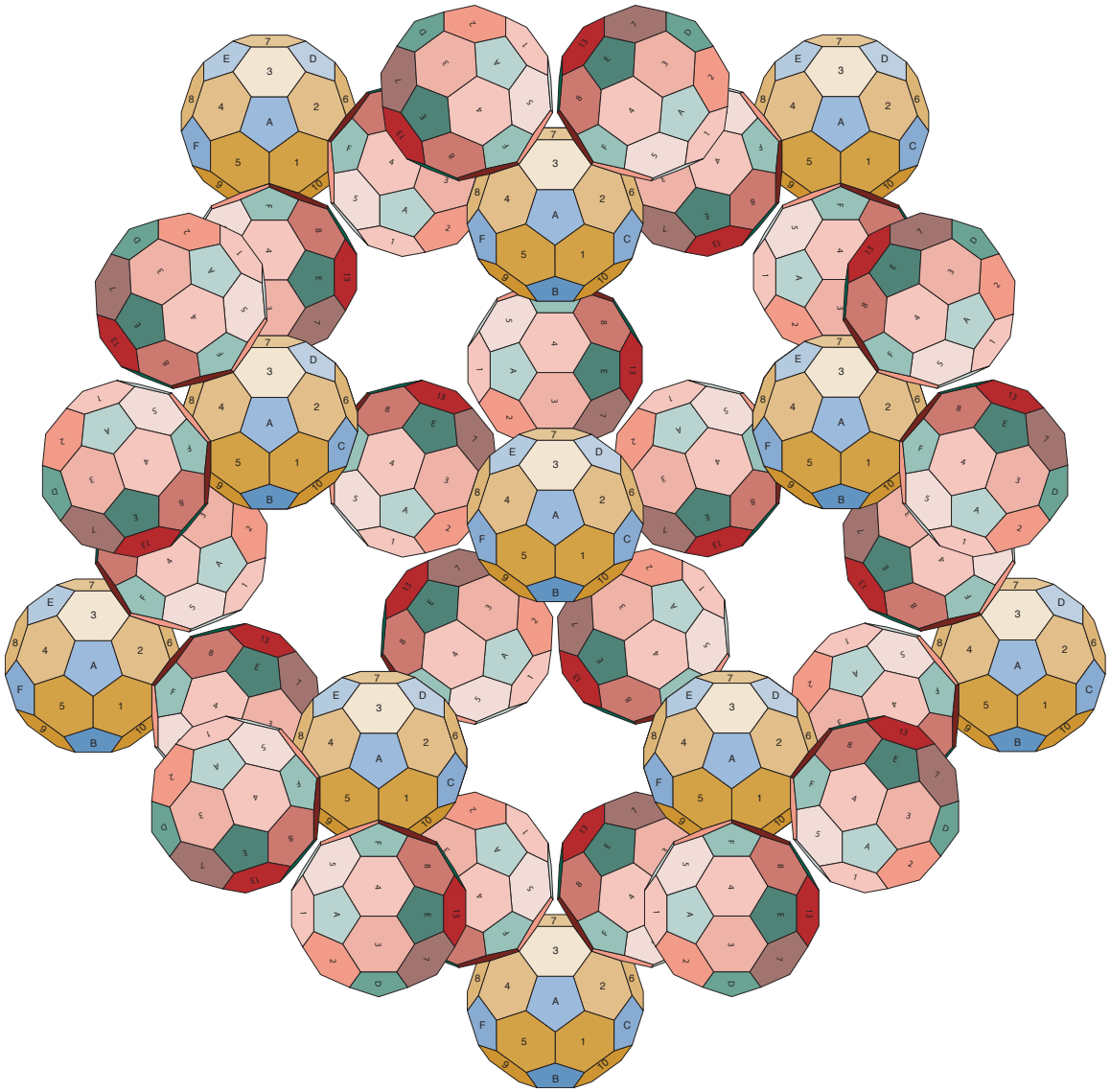


Fig. 55 Pentagonal dodecahedral assembly–lower assembly

The lower mid-layer has been joined with the bottom hub to complete the lower assembly shown in the figure.

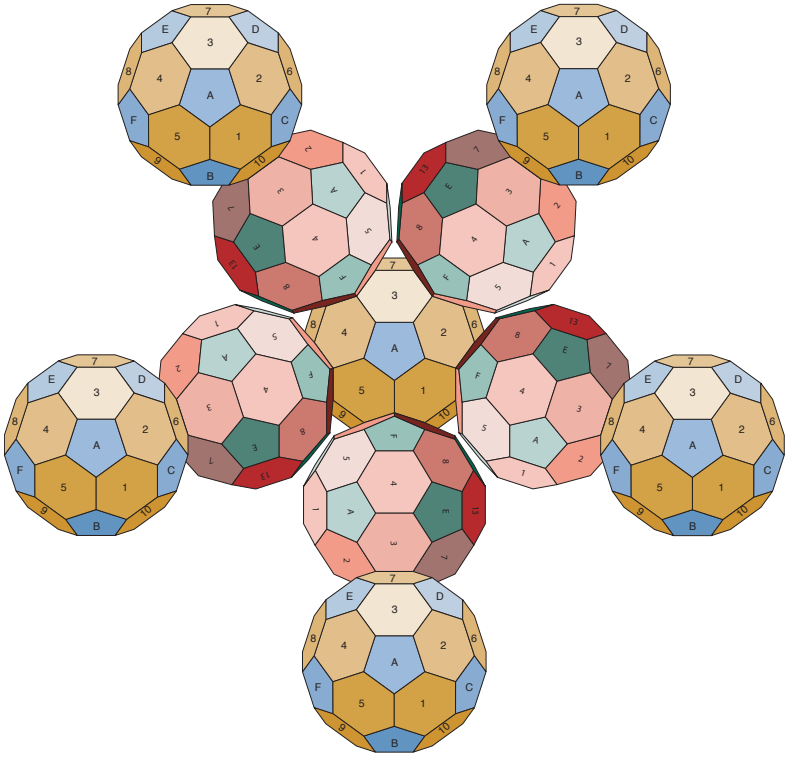


Fig. 56 Pentagonal dodecahedral assembly–top hub
The figure shows the top hub of the upper assembly with a base unit attached to each of it β-joiners.

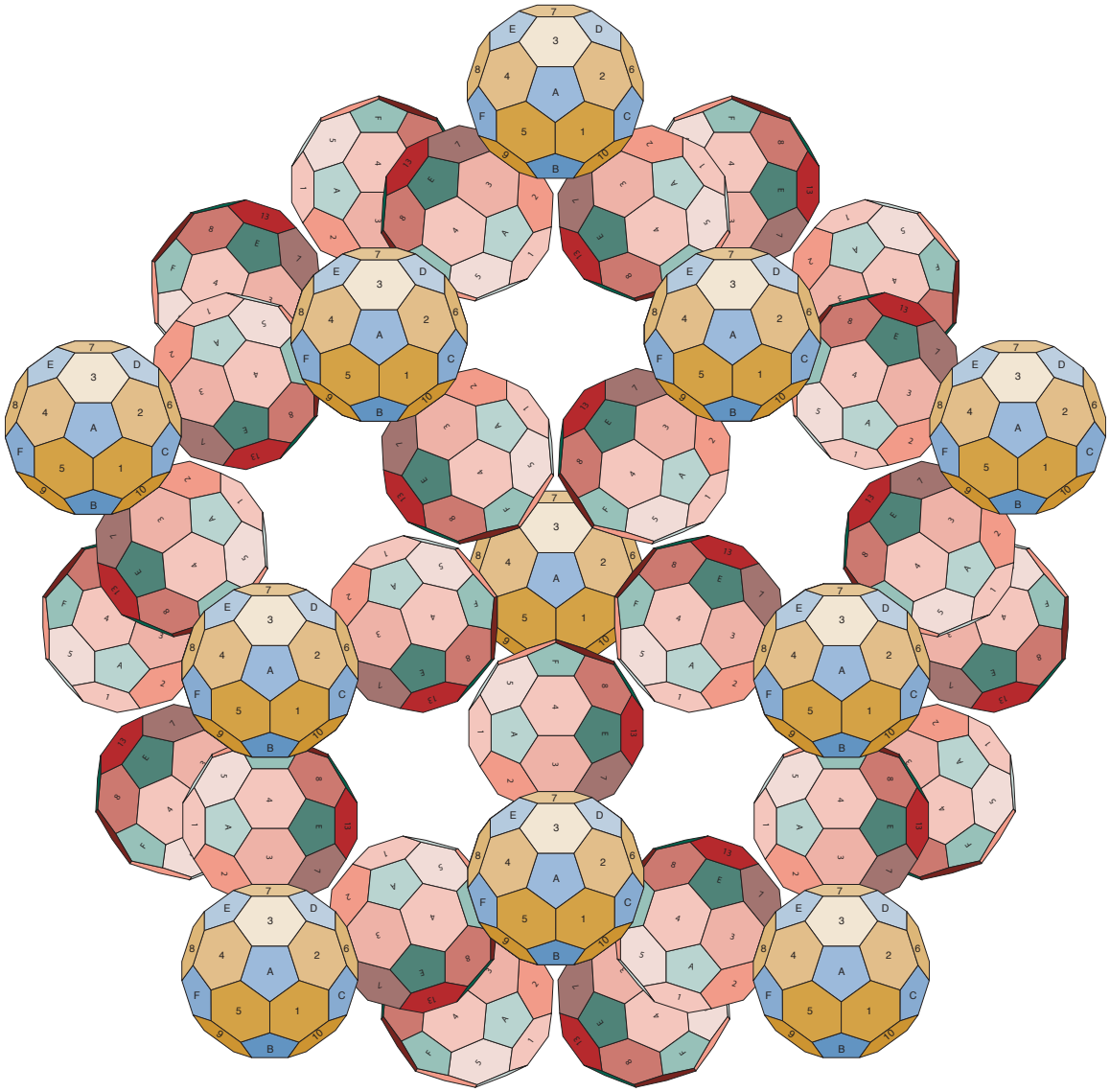


Fig. 57 Pentagonal dodecahedral assembly–upper assembly
The figure shows the upper assembly formed by the joining of the upper mid-layer and the top hub.

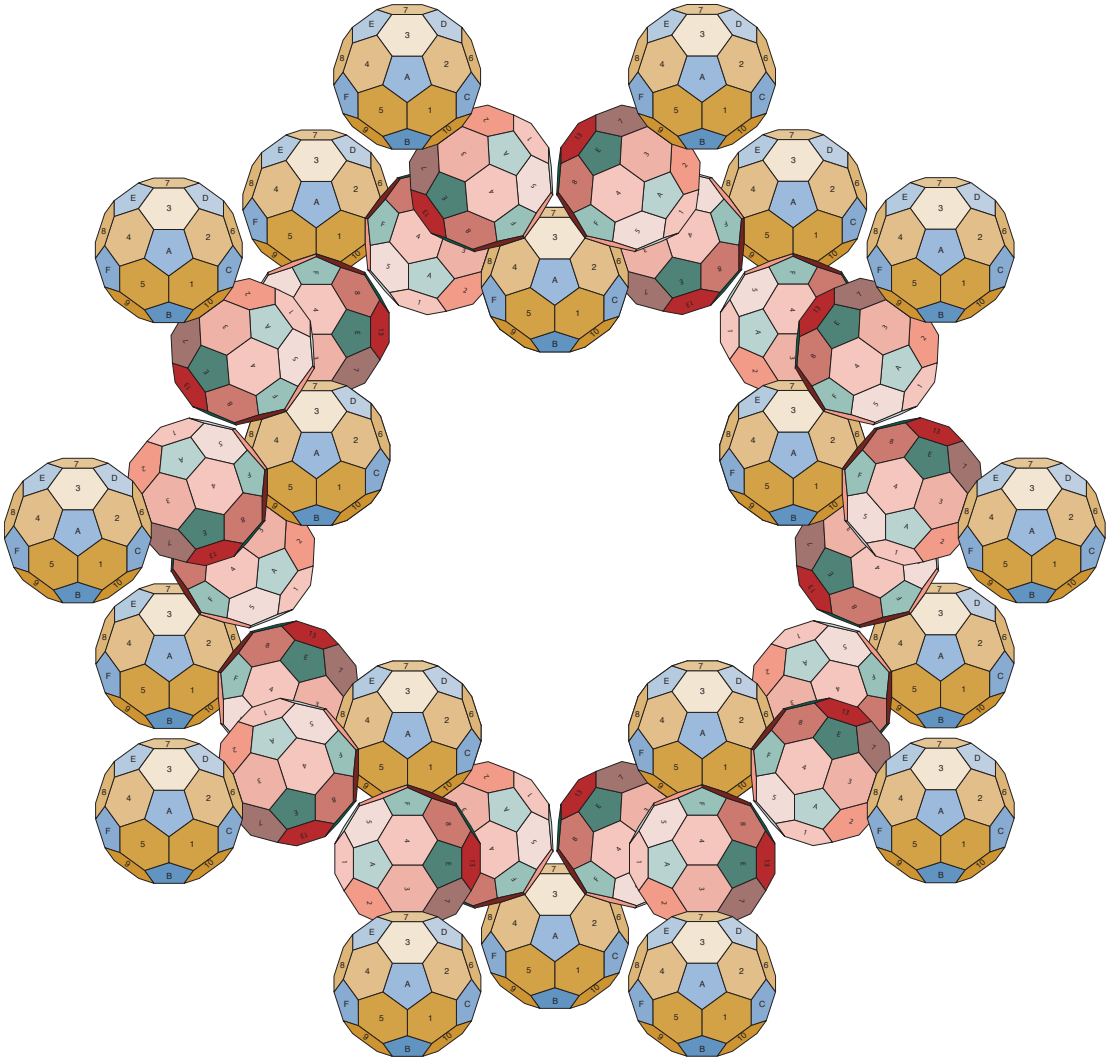


Fig. 58 Pentagonal dodecahedral assembly–lower mid-layer with linking base units

The figure shows the lower mid-layer with the ten base units required to join it with the upper mid-layer. Two are attached to each of the five hubs. The additional base units are radially outermost.

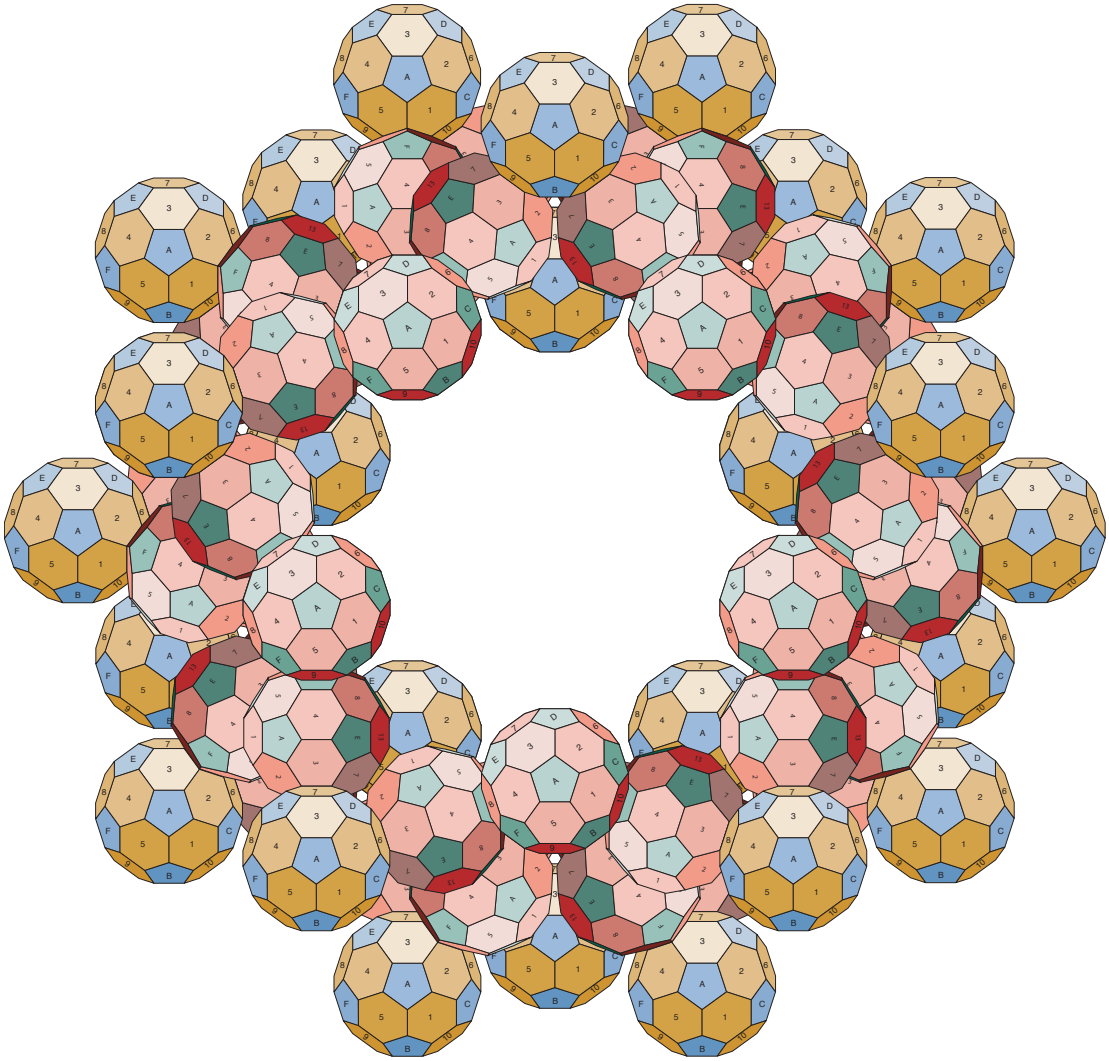


Fig. 59 Pentagonal dodecahedral assembly–assembly of mid-layers
The figure shows the upper and lower mid-layers combined in a single assembly.

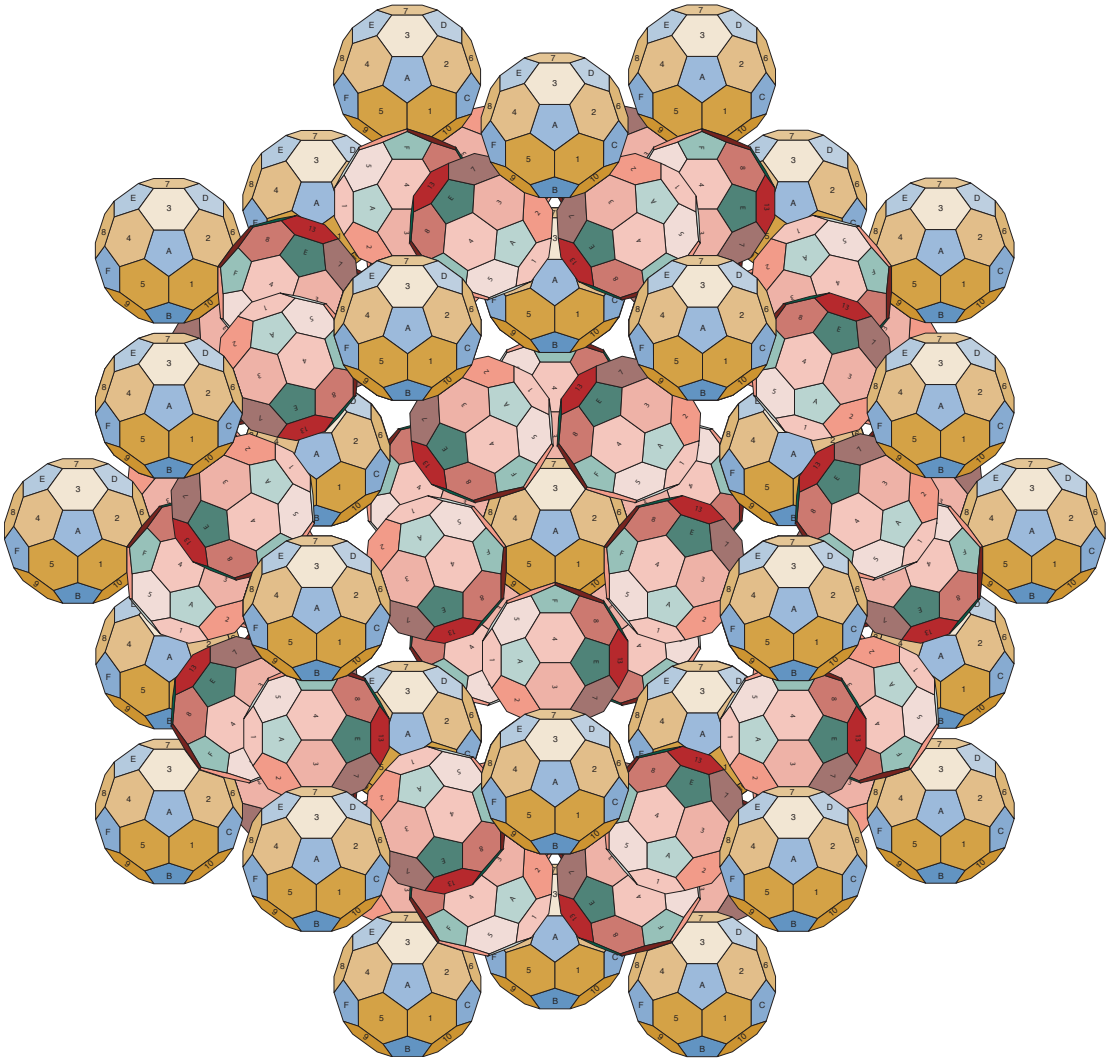


Fig. 60 Pentagonal dodecahedral assembly–complete

The figure shows the completed pentagonal dodecahedral assembly–upper and lower mid-layers with top and bottom hubs.

Triacontahedron

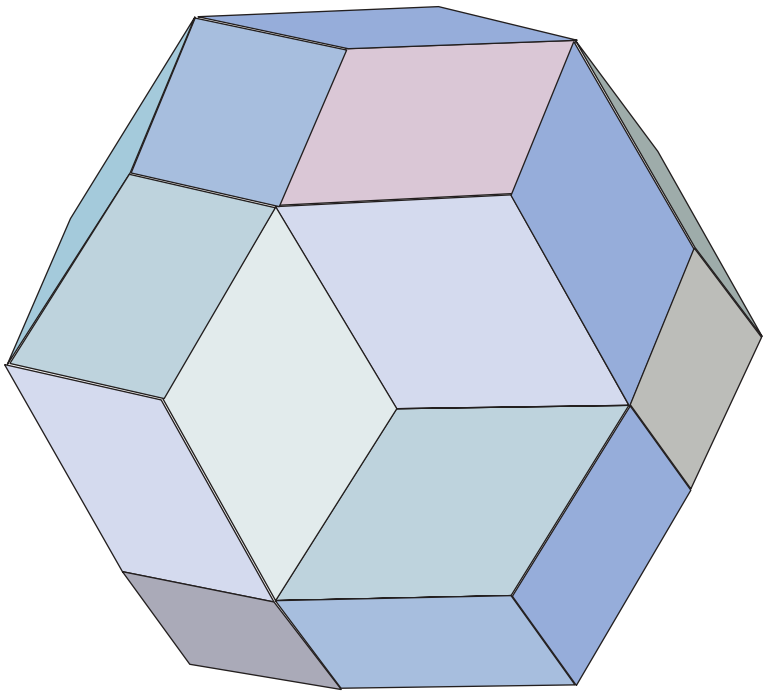


Fig. 61 Triacontahedron–oblique view

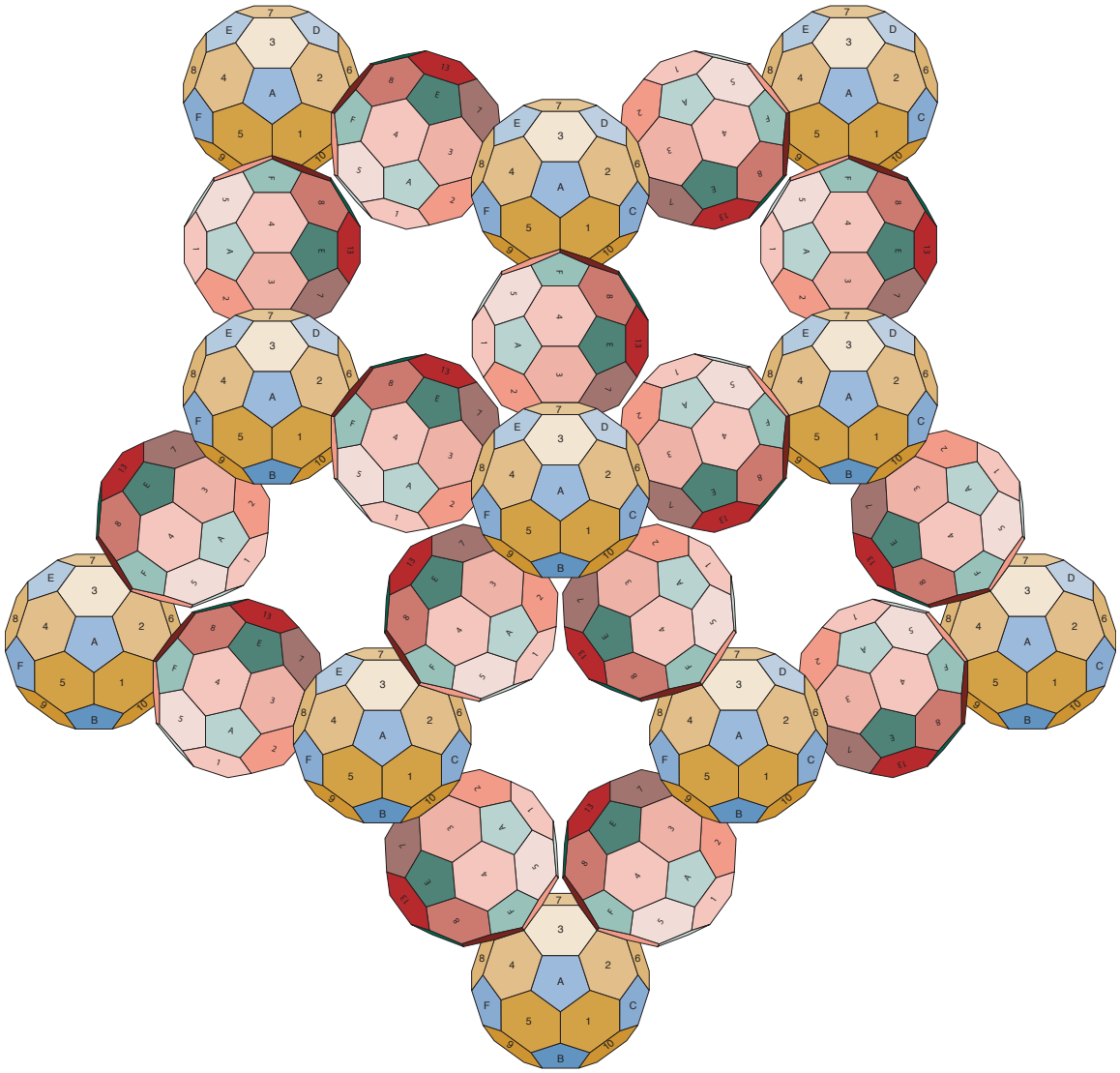


Fig. 62 Triacontahedron–five faces at the top pentagonal vertex

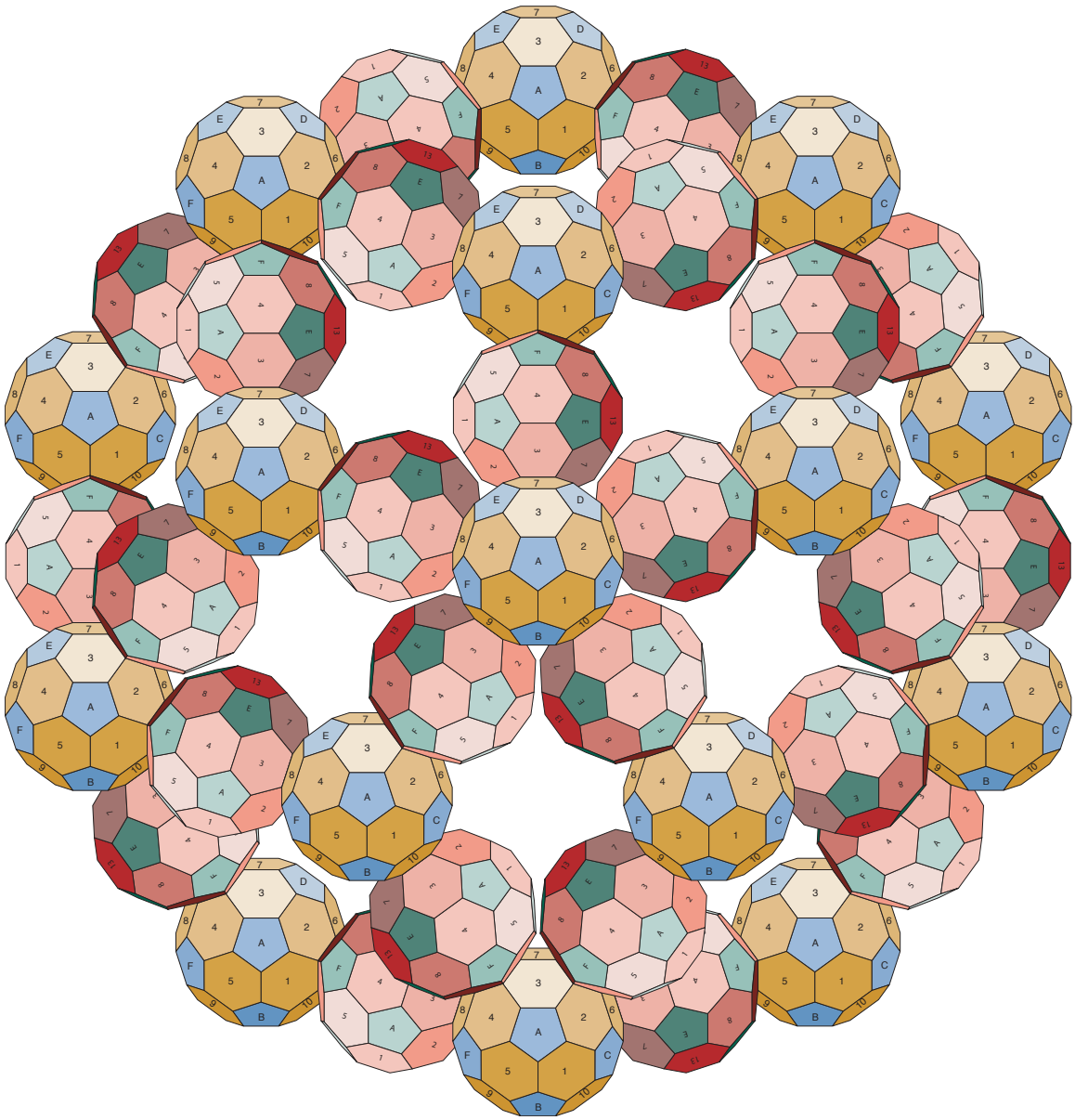


Fig. 63 Triacontahedron–ten faces at the top pentagonal vertex

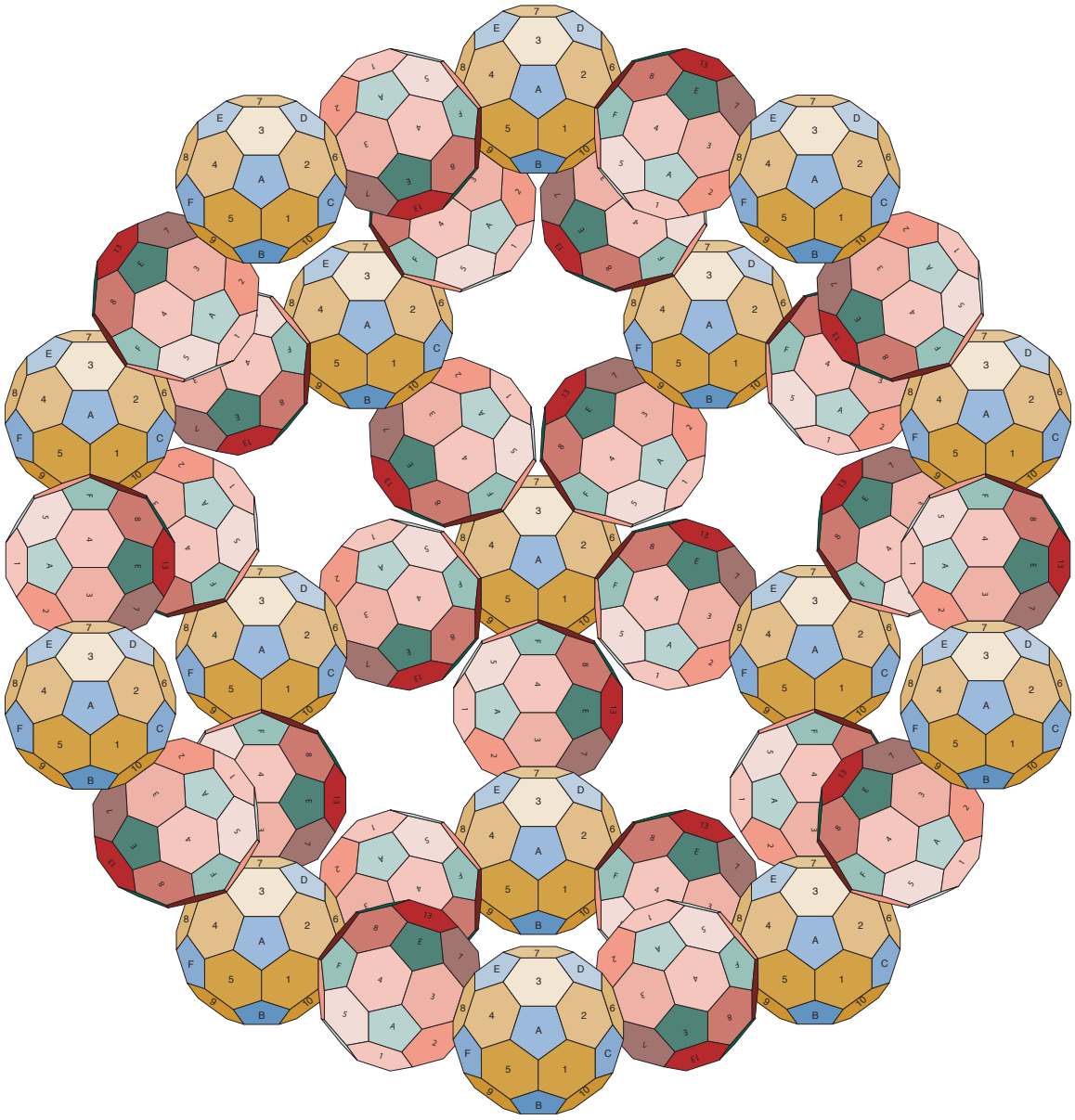


Fig. 64 Triacontahedron–ten faces at the bottom pentagonal vertex

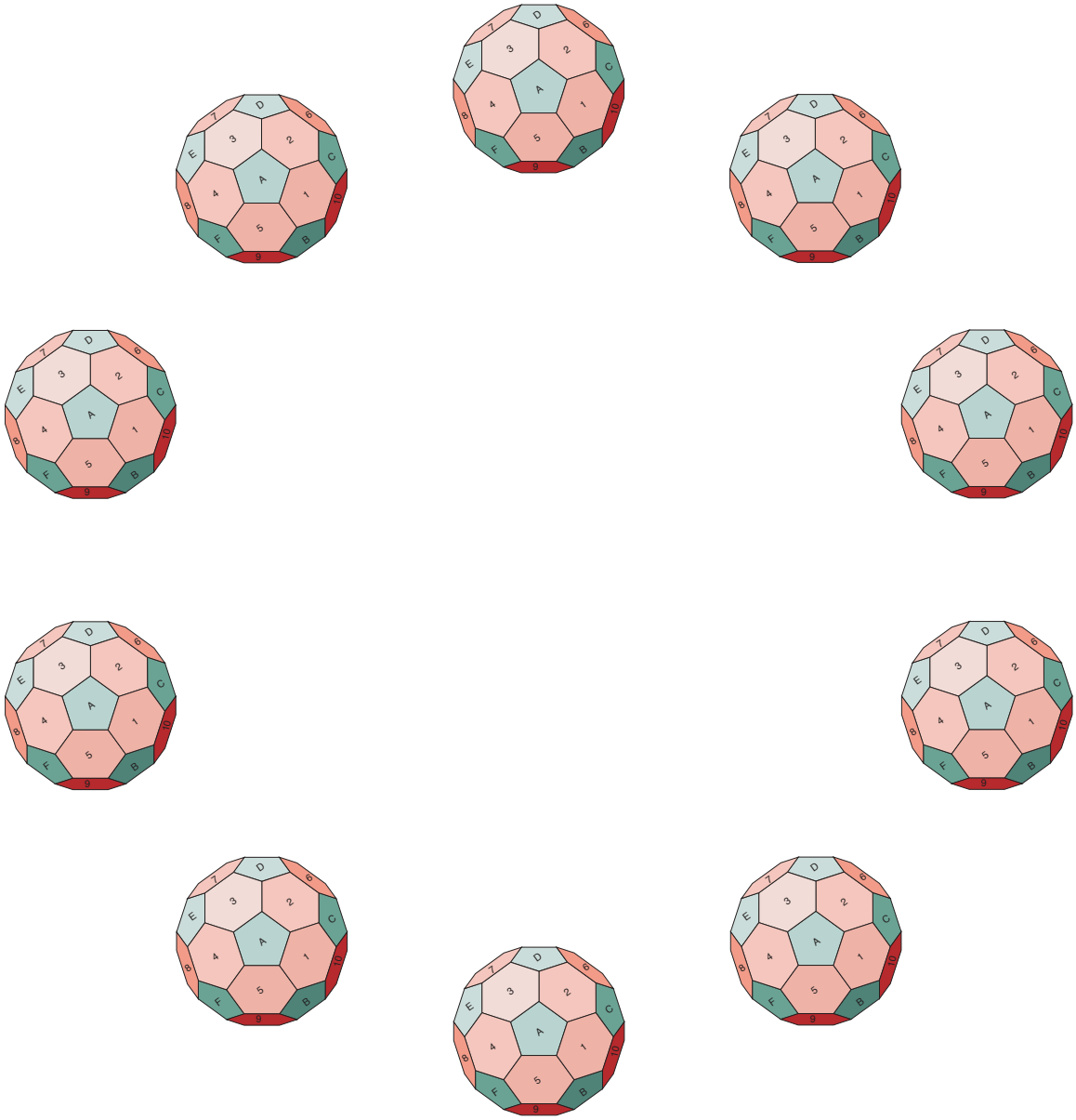


Fig. 65 Triacontahedron–mid-layer
The mid-layer of the triacontahedral assembly consists of ten joiner units. Each of these units joins to a base unit of both the lower and upper assemblies.

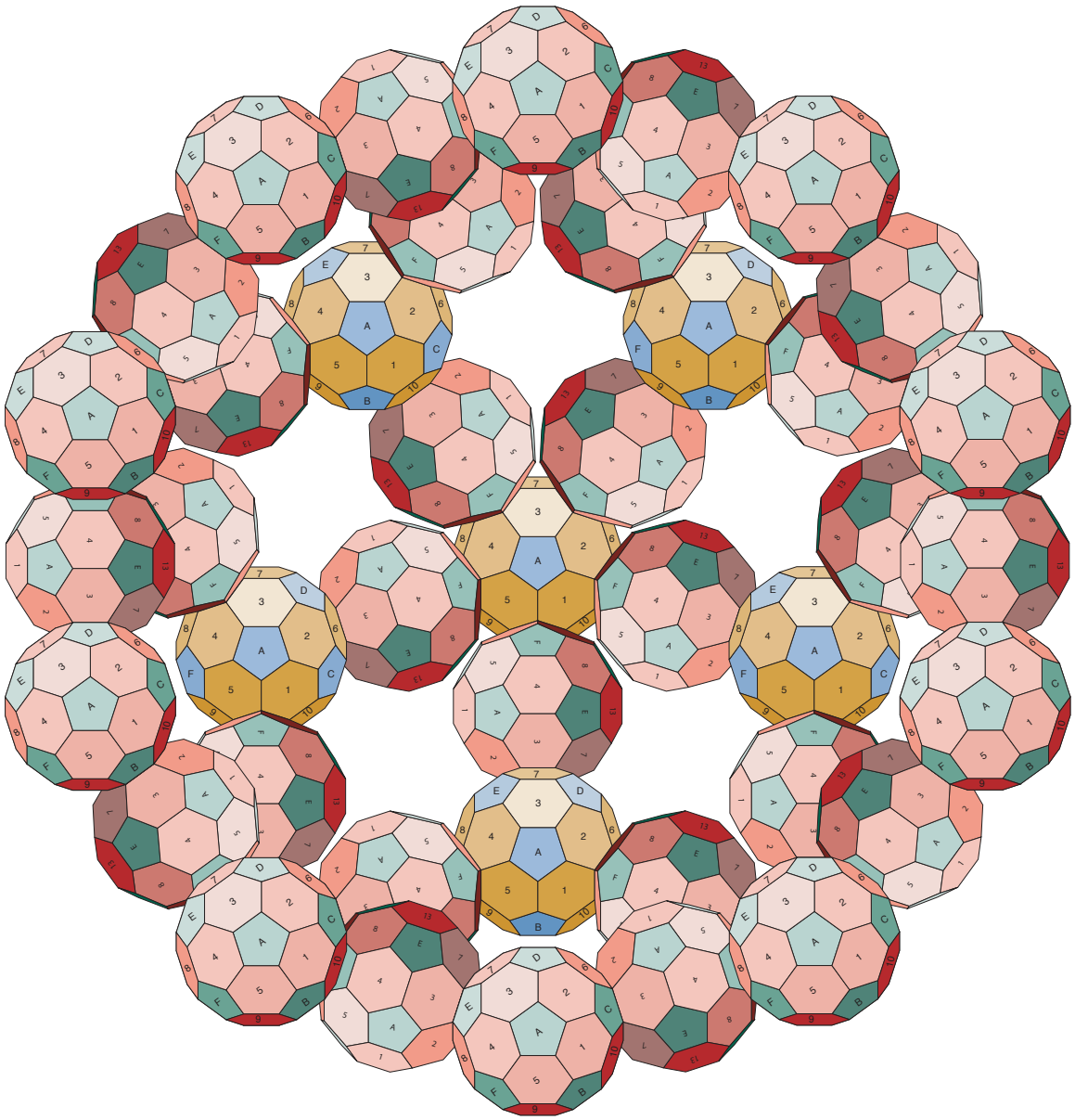


Fig. 66 Triacontahedron–bottom ten faces plus mid-layer
The figure shows the ten units of the mid-layer adjoined to the base units of the lower assembly.

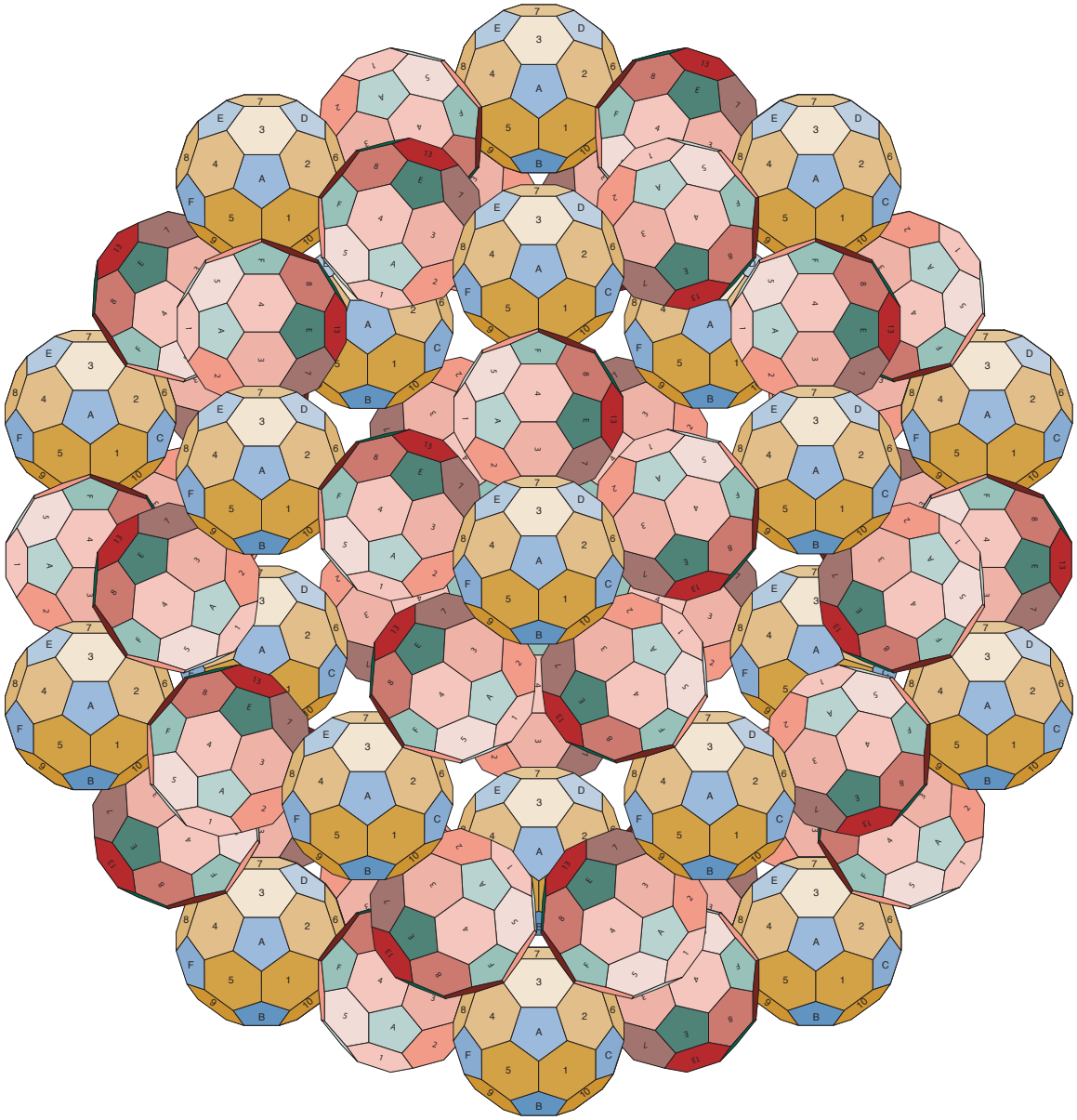


Fig. 67 Triacontahedron–complete
With the lower, middle, and upper layers in place, the triacontahedral assembly is complete.

Elongated triacontahedron

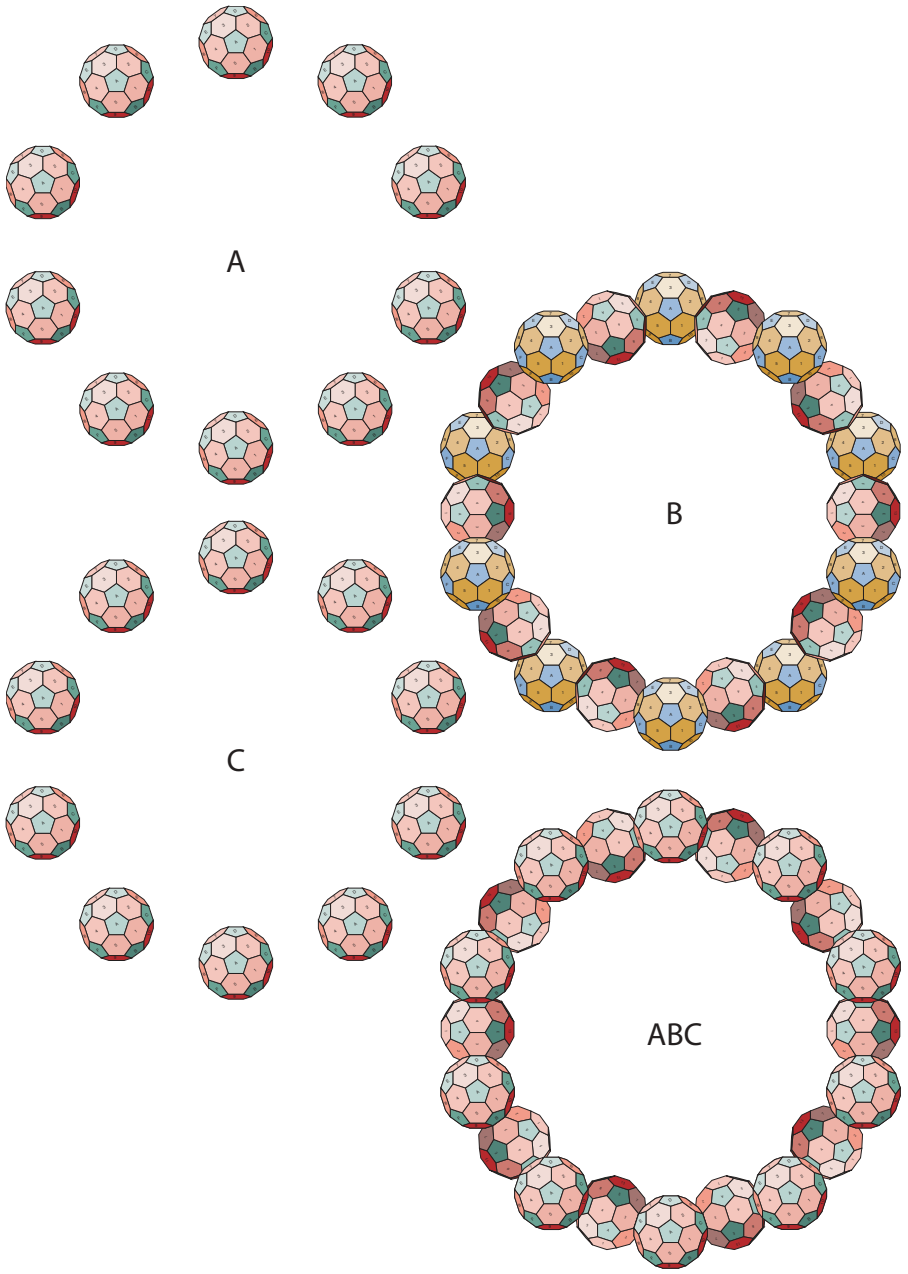


Fig. 68 Elongated triacontahedron–assembly of the middle layer

The figure shows the components of the middle layer of the elongated triacontahedron. Rings A and C are identical. Ring B is a structural assembly twenty icosidodecahedra–ten base units with ten β-joiners. Each of the joiner units of ring A join to the L face of a base unit of ring B; each of the joiner units of ring C join to the A face of a base unit of ring B. The complete assembly is labeled ABC.

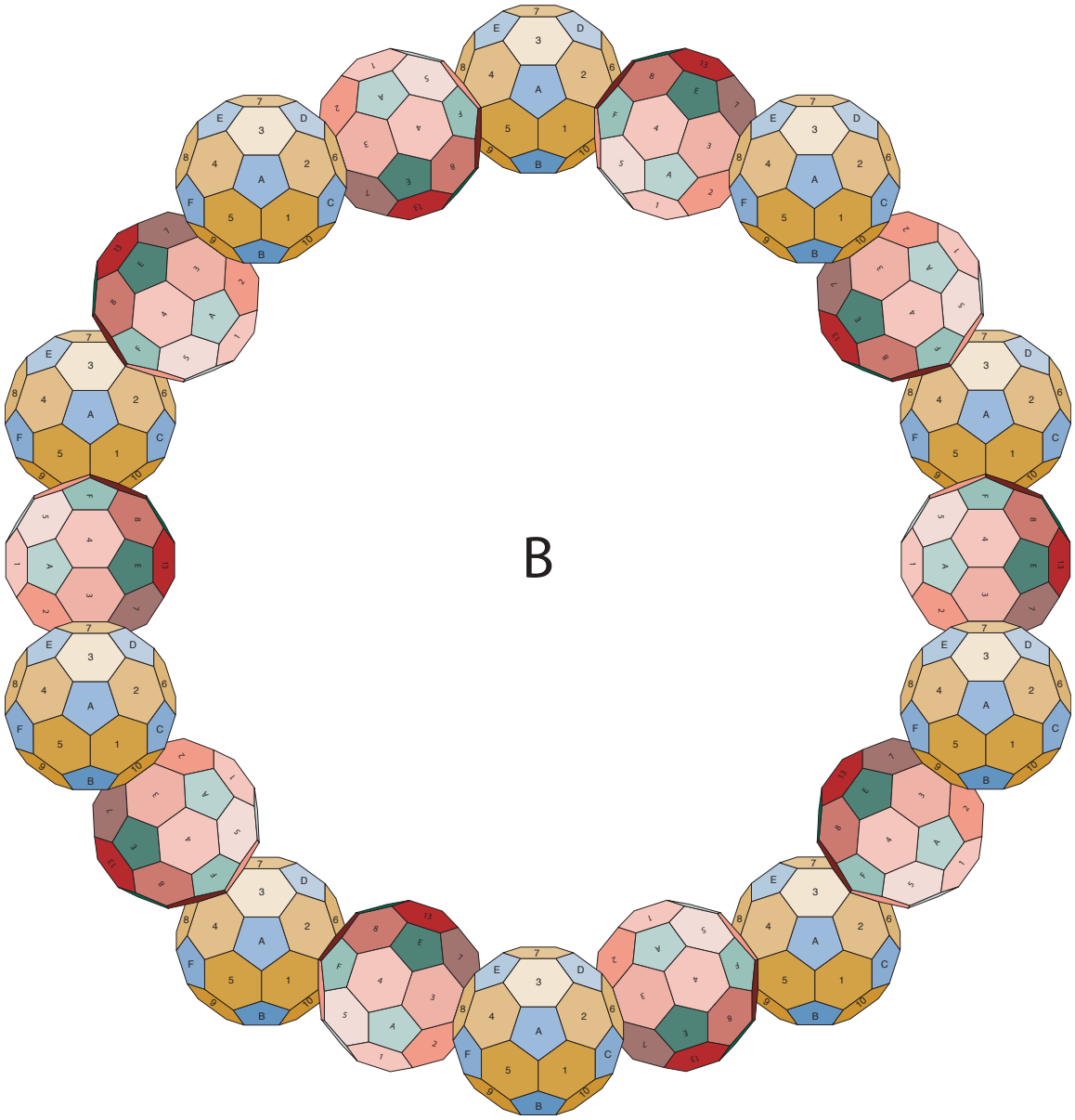


Fig. 69 Elongated triacontahedron–ring B of the middle layer

The figure shows ring B of the middle layer of the elongated triacontahedron. The ring consists of alternating base and joiner units.

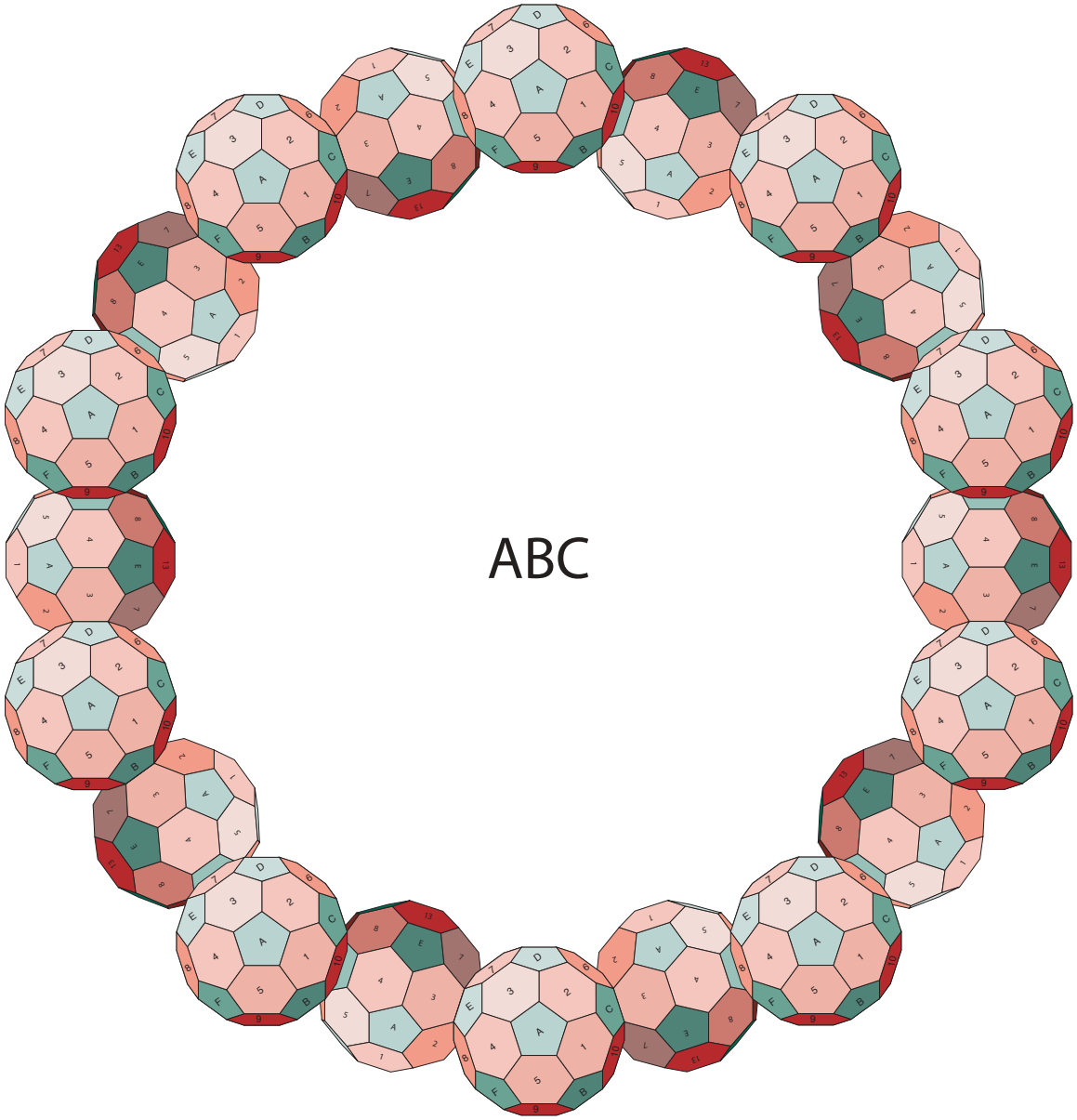


Fig. 70 Elongated triacontahedron–middle layer

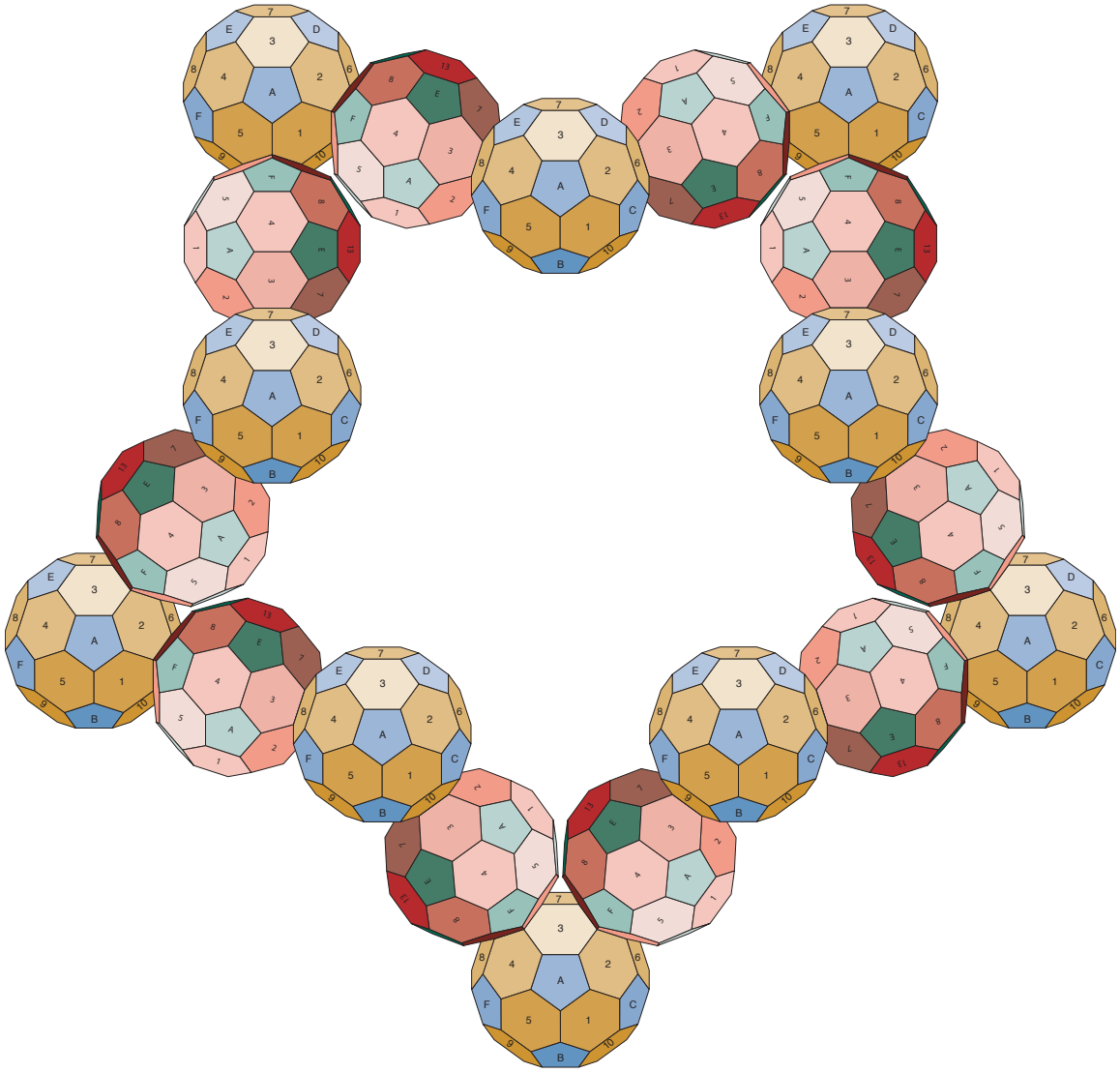


Fig. 71 Pentagonal ring consisting of ten base units and ten joiners

The figure shows a ring of twenty hexagonal face icosidodecahedra joined pentagonal face to pentagonal face. The ring occurs in the icosahedral assembly of γ-hubs shown earlier in this document. [See Figures 38 and 41.]

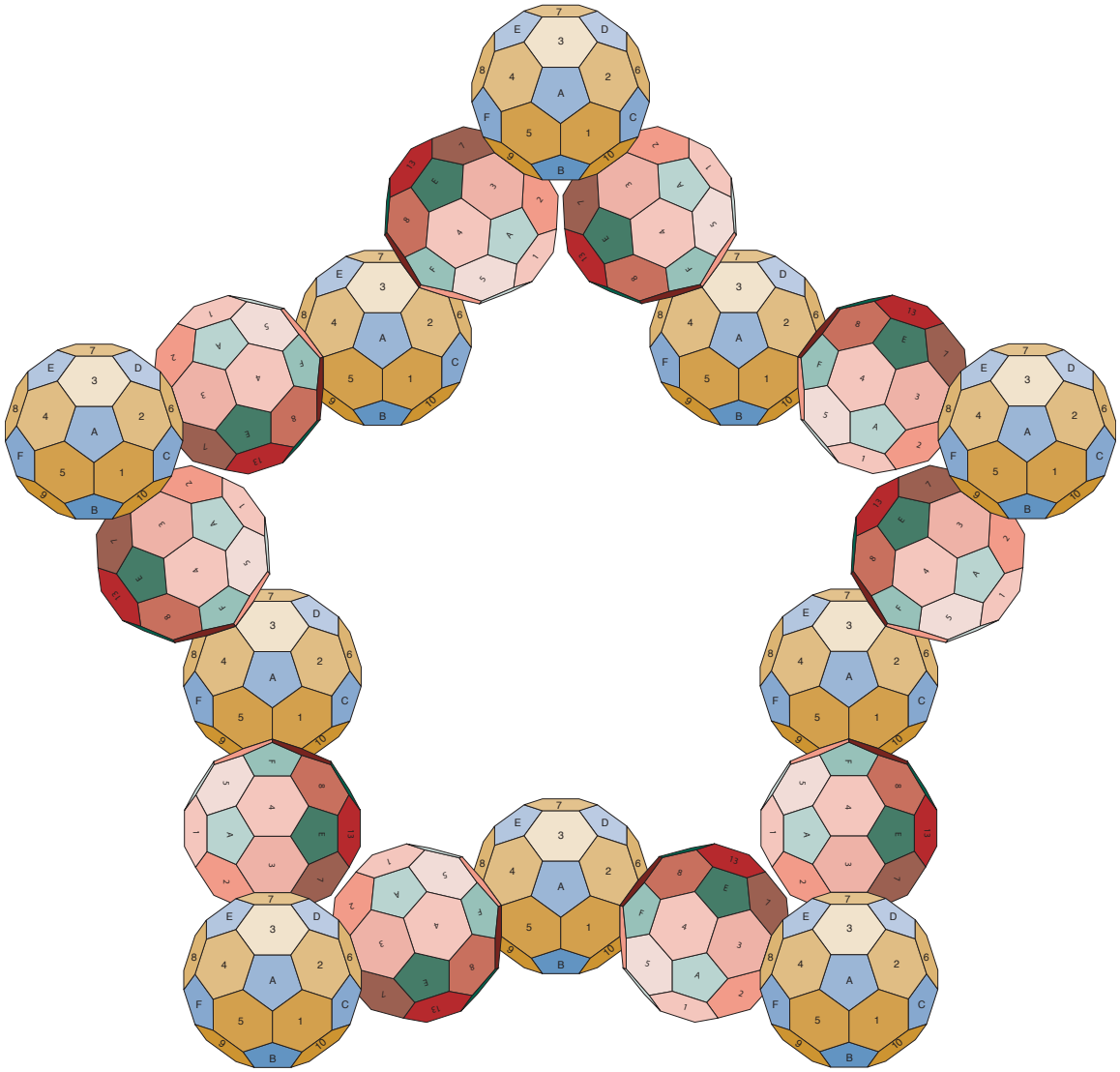


Fig. 72 Pentagonal ring consisting of ten base units and ten joiners–obverse